Accelerated Dempster-Shafer using Tensor Train Decomposition*

Cassandra Armstrong CLARMSTRONG@LANL.GOV
Los Alamos National Laboratory, Space Science and Applications Group

Duc P. Truong DPTRUONG@LANL.GOV
Erik Skau EWSKAU@LANL.GOV
Kari Sentz KSENTZ@LANL.GOV
Los Alamos National Laboratory, Information Sciences Group

Dempster-Shafer (DS) theory is a general framework for reasoning with uncertainty [1, 3]. Traditional DS algorithms work on a *frame of discernment* or a collection of propositions. A *basic probability assignment* (BPA) is a function that assigns non-negative masses to each element of the powerset of the frame of discernment. All BPA masses must sum to one, and in DS theory the mass assigned to the emptyset must be zero. BPAs are used to represent uncertainties in the propositions and can be combined together with rules of combination such as the *conjunctive join* or *Dempster's rule of combination*. *Belief* and *plausibility* are two measures of uncertainty that provide lower and upper bounds respectively for the probability that a specific hypothesis is provable. One practical problem with DS implementations is that the complexity of these operators is exponential in the number of propositions. Specifically, with N propositions in the frame of discernment, the complexity of DS operators can be as bad as $O(2^{3N})$. In this work, we explore the use of tensor decompositions to accelerate the computations of Dempster's rule of combination, belief, and plausibility and to overcome the curse of dimensionality.

Recently, the tensor train (TT) decomposition has been shown to provide a useful low-rank approximation framework for high dimensional problems [2]. Here we present a novel approach using TT decomposition to overcome the curse of dimensionality and accelerate the traditional DS computations. We first show that DS operators observe explicit low-rank representations in the TT format. With this observation, the DS computations can be highly accelerated by applying the TT format DS operators to specific reshapes of the BPAs. Secondly, we further propose to approximate BPAs with TT decomposition and describe the algorithms necessary to apply DS operators to TT BPAs. With TT decompositions we decrease the exponential complexity to be linear in N under certain circumstances.

Additionally, we investigate how the error from BPA approximation propagates through DS operators. Under the assumptions that the approximations of BPAs are non-negative and sum-to-one, we provide tight upper bounds on the error propagation. Namely, if $||m_1 - \hat{m}_1||_1 \leq \epsilon_1$, and $||m_2 - \hat{m}_2||_1 \leq \epsilon_2$, then

$$\begin{aligned} |\,\mathrm{belief}_{m_1}(x) - \mathrm{belief}_{\hat{m}_1}(x)| &\leq \epsilon_1 \\ |\,\mathrm{plausibility}_{m_1}(x) - \mathrm{plausibility}_{\hat{m}_1}(x)| &\leq \epsilon_1 \\ ||(m_1 \oplus m_2) - (\hat{m}_1 \oplus \hat{m}_2)||_1 &\leq \epsilon_1 + \epsilon_2 + \epsilon_1\epsilon_2 \ . \end{aligned} \tag{1}$$

Lastly, we demonstrate the superior computational performance of our accelerated Dempster-Shafer data analysis on synthetic data. We also discuss potential extensions of this work to other methods of imprecise probability, such as Transferable Belief Model (TBM) and Generalized Evidence Theory (GET).

References

[1] Arthur P. Dempster. Upper and lower probabilities induced by a multivalued mapping. *The Annals of Statistics*, 28: 325–339, 1967.

[2] Ivan V Oseledets. Tensor-train decomposition. *SIAM Journal on Scientific Computing*, 33(5):2295–2317, 2011.

[3] Glenn Shafer. *A Mathematical Theory of Evidence*, volume 42. Princeton university press, 1976.

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