Measuring Deviation from Stochastic Dominance under Imprecision

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Stochastic orders [2, 5] have been successfully used in probability and statistics and in many related fields of application for the comparison of random variables. The most classical stochastic order is that of stochastic dominance. Formally, we say that a random variable stochastically dominates another random variable if the cumulative distribution function of the first random variable is pointwisely smaller than or equal to the cumulative distribution function of the second random variable. Among many other intuitive properties, the fact that a random variable stochastically dominates another random variable implies that the expected value of the first random variable is greater than or equal to the expected value of the second random variable, the converse implication being true within a location distribution family.

Unfortunately, stochastic dominance does not necessarily hold in practice. For instance, within a location-scale family generated from a standard distribution with unbounded support (such as the Gaussian distribution family), there do not exist two random variables with different variance such that one of the random variables stochastically dominates the other. Still, if the location parameters of both random variables are very far from one another, this stochastic dominance would be 'very close' to being satisfied. For this reason, recent work has proposed the so-called γ-index [1] for measuring how far a random variable is from stochastically dominating another. Interestingly, this γ-index can be used for defining a statistical hypothesis test for approximate stochastic dominance, i.e, testing whether stochastic dominance holds up to some small contamination in the model.

In the context of imprecise probabilities, the problem of studying stochastic dominance between sets of distributions arises. Several generalizations of the notion of stochastic dominance to this setting have been proposed [4]. In particular, we devote our attention to (FSD_1) stochastic dominance, which means that all distributions in the first set of distributions stochastically dominate all distributions in the second set of distributions, and (FSD_4) stochastic dominance, which means that there exists a distribution in the first set of distributions that stochastically dominates at least one distribution in the second set of distributions.

In the present work, we go further and explore how the γ-index could be defined for the comparison of sets of distributions. In particular, we introduce two generalizations of the γ-index for measuring how far a set of distributions is from stochastically dominating another, one in the sense of (FSD_1) stochastic dominance and another in the sense of (FSD_4) stochastic dominance. Special attention is paid to the case in which the sets of distributions are obtained under two different circumstances: (i) there is imprecision in the value of the parameter of a parametric distribution family and only an interval of possible values is available; and (ii) there is imprecision in the whole distribution and only a p-box is available. We explore this approach under different forms of dependence between the distributions. The obtained results are of interest to many fields of application such as social welfare (see, e.g., [4]) and decision making (see, e.g., [3]).

References

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