Modelling Higher Order Evidence with Second Order Imprecise Probability

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Agents sometimes obtain higher order evidence about their own rational capacities, epistemic performance, or evidential situation in their reasoning with first order evidence [2]. For simplicity, I refer to the content of higher order evidence as regarding the agent's irrationality. The epistemological questions are: what the agent's credence should be in her first order belief, and in her higher order belief that she is irrational in her reasoning for her first order belief. The Level Splitting Interpretation argues that the agent's first order belief should only respond to her first order evidence [1], since the higher order evidence bears no relevance to the content of the first order belief. For parallel reason, the agent's higher order belief should only respond to her higher order evidence. Therefore, the agent will be in the state of epistemic akrasia, where her higher order belief diverges from her first order belief.

Adopted from Lyon's project [3], I propose using second order Imprecise Probability as a descriptive model to represent agents' credence in epistemic akrasia. Other authors have discussed ways in which second order Imprecise Probability are philosophically significant. Jonas Mork, David Sundgren and Alexander Karlsson have written on the formal methods for measuring uncertainty in first and second order probability distributions. Robert Nau has written on the formal method of first and second order expected utility calculation for differentiating between uncertainty and risk in Ellsberg's experiment. My philosophical project concerns a different epistemic problem: modelling an agent's opinions about the possible rational responses to her first order evidence. The second order Imprecise Probability model I employ is correspondingly different, though could perhaps be improved by incorporating elements of the aforementioned frameworks.

Consider an example [2]. There is a clock on the wall that has no marks nor numbers. It has only one minute hand which jumps discretely from one minute to the next. I take a look at the clock but cannot tell where exactly the hand is. However, I should have the highest credence in where I believe it most likely is and lower credence in the two positions next to it. Based on my vision, I believe with credence $[0.4, 0.6]$ that the hand is pointing at minute 20 in the hour. I also believe with credence $[0.2, 0.3]$ in the hand is pointing at 19, and the same credence $[0.2, 0.3]$ in 21. I consider no other possibilities. Upon reflection, my observation also has the role of higher order evidence which bears on the irrationality of my first order belief. Suppose if my credence in minute 20 is 0.6, then I have credence 0.4 in that the hand is at 19 or 21. But if those are the case, then I should have the highest credence in 19 or 21 and my credence in 20 is too high. Therefore, I should have credence 0.4 in that my first order belief is irrational. The equation of the relation between my first order belief (x) in 20 and my higher order belief (y) of its irrationality is then $y = 1 - x$, $x \in [0.4, 0.6]$ $y \in [0.4, 0.6]$. This equation of my mental state in higher order doubt forms a second order Imprecise Probability that cannot be captured by first order Imprecise Probability. Consider my other first order belief (x') in the hand is at either 19 or 21 and my higher order belief (y') of its irrationality. The first order Imprecise Probability is also $x' \in [0.4, 0.6]$ $y' \in [0.4, 0.6]$, which is the same as that of my belief in 20. But the equation of the relation between the first and higher order belief is the different second order Imprecise Probability $y' = x'$. This example is generalisable and other cases of higher order evidence would require other second order Imprecise Probability equations to depict the mental states of the agents. In general, the agent's first order belief is represented by a set of points on the number line $X = \{x'' \mid 0 \leqslant x'' \leqslant 1 \& F\}$, where F is the condition for x'' derived from the first order evidence. The agent's higher order belief is represented by $Y = \{y'' \mid 0 \leqslant y'' \leqslant 1 \& H\}$, where H is the condition for y'' derived from the higher order evidence. The agent's local mental state of epistemic akrasia is represented by a set of points on the plane $\Omega = \{(x'', y'') \mid 0 \leqslant x'' \leqslant 1 \& F \& 0 \leqslant y'' \leqslant 1 \& H \& R\}$ $(\Omega \subseteq X \times Y)$, where R is the relation between the first and higher order belief, derived from the agent's overall evidence. The agent's complete epistemic state of opinions is modelled by a set of such probability functions.

References

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[3] Aidan Lyon. Vague Credence. *Synthese*, 194(10):3931–3954, 2017. doi:10.1007/s11229-015-0782-5.