Desirable Gambles Based on Pairwise Comparisons*

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This abstract presents some ongoing research in a particular case of coherent sets of desirable gambles [1]: the sets that are generated by pairwise comparisons. If X is a reference set (the possibility space), then a pairwise comparison is a gamble of the form $a_1 I_{x_1} + a_2 I_{x_2}$, where $x_1, x_2 \in X$, $a_1, a_2 \in \mathbb{R}$, and I is the indicator function, i.e. we are expressing our preference of x_1 against x_2. A gamble like this only makes sense when $a_1.a_2 < 0$, as in other cases, it will be trivial or will imply a sure loss. Without loss of generality, we will assume that $a_1 = 1, a_2 < 0$. One example of this kind of set is discussed in [3], where the uniform model is defined when all gambles $I_{x_1} - aI_{x_2}$ where $a < 1$ are desirable.

An initial specification will be given by a mapping $t : X \times X \longrightarrow \mathbb{R} \cup \mathbb{R}^- \cup \{+\infty\}$, where $\mathbb{R}^- = \{a^- \ : \ a \in \mathbb{R}\}$, $t(x_1, x_2) \geq 0, t(x, x) = 1^-$. The intuitive meaning of a^- is any value strictly lower than a. If $t(x_1, x_2) = a$, this will mean that $I_{x_1} - aI_{x_2}$ is desirable. In the finite case, $X = \{x_1, \ldots, x_n\}$, a specification t can be represented as an $n \times n$ matrix, T, where the element $T_{ij} = t(x_i, x_j)$.

In this poster, we will discuss how to compute natural extension, how to check avoiding sure loss, how to discount a set of gambles, and the relationships of these specifications with multiplicative preference relationships.

Natural Extension. The natural extension of a specification t can be computed by considering the following sequence of functions:

$$t^1 = t, \qquad t^{i+1} = \max\{t^i \circ t, t^i\},$$

where $t \circ g(x_1, x_2) = \max_{x \in X} t(x_1, x).g(x, x_2)$, taking into account that if the supremum of values $t(x_1, x).g(x, x_2), x \in X$ is a but a is not equal to any value $t(x_1, x).g(x, x_2)$ then the maximum will be a^-, and that the maximum is pointwise applied. $\{t^i\}_{i \in \mathbb{N}}$ is an increasing sequence, and the pointwise limit will be called t^*. Then, the initial specification avoids sure loss if and only if $t^*(x_i, x_i) = 1^-$ for any $x_i \in X$. In that case, $I_{x_1} - aI_{x_2}$ belongs to the natural extension of the initial specification when $a \leq t^*(x_i, x_j)$.

In the finite case, the functions t^i converge after n iterations, i.e. $t^{i+1} = t^i$, for $i \geq n$. The limit can be computed as a problem of finding the path with maximum cost in a weighted graph. The natural extension of a generic, non-necessarily binary, gamble f can be solved as a max flow problem with gain/loss factors.

Discounting. Moral [2] gives a definition of discounting was given generalizing the ϵ-discounted models. Given a coherent set of desirable gambles $\mathcal{D}$ the ϵ-discounting was the set of gambles $t - \epsilon \inf t$ where $t \in \mathcal{D}$. Discounting a set of gambles based on pairwise comparisons with natural extension given by t^* is very simple: we have to compute $t^{*\epsilon}(x_i, x_j) = t^*(x_i, x_j)(1 - \epsilon)$.

Preference Relationships. A pairwise comparison will be called maximal if $t^*(x_i, x_j).t(x_j, x_i) = 1^-$ for any $x_i, x_j \in X$. In that case, there is an obvious relationship with multiplicative preference relationships [4, Sect. 3] that will be discussed in the poster.

References

[1] Inés Couso and Serafín Moral. Sets of desirable gambles: conditioning, representation, and precise probabilities. *International Journal of Approximate Reasoning*, 52(7):1034–1055, 2011.

[2] Serafín Moral. Discounting imprecise probabilities. In *The Mathematics of the Uncertain: A Tribute to Pedro Gil*, pages 685–697. Springer, 2018.

[3] Serafín Moral. Learning with imprecise probabilities as model selection and averaging. *International Journal of Approximate Reasoning*, 109:111–124, 2019.

[4] Thomas L Saaty. *Fundamentals of decision making and priority theory with the analytic hierarchy process*. RWS publications, 1994.

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