A Belief Model for Conflicting and Uncertain Evidence: Connecting Dempster-Shafer Theory and the Topology of Evidence*

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The focus of our research is on the problem of combining uncertain and potentially conflicting evidence from an interdisciplinary perspective. This problem can be explored from different views, and we are considering two of them as a base: Dempster-Shafer Theory (DST) of belief functions [2], a mathematical theory to combine uncertain evidence; and Topological Models of Evidence (TME) [1], an epistemic approach to combine possibly mutually inconsistent evidence.

We aim to bring insights into this problem by combining methods from these two approaches. Our first contribution is a new model for measuring degrees of beliefs based on a body of possibly mutually inconsistent, incomplete, and uncertain pieces of evidence which also allows agents to have different evidential demands depending on the context. We will consider as input a set S of possible states, a set $\mathcal{E} \subseteq 2^S$ of pieces of evidence, and a nonempty set $\mathcal{E}^Q \subseteq \mathcal{E} \times (0,1)$ which contains pairs of a piece of evidence and its degree of certainty. From the perspective of DST, each pair $(E, p) \in \mathcal{E}^Q$ can be read as a mass function m with focal element E such that $m(E) = p$. We will adopt the axiomatic definition of belief functions from this theory, and the output of our method will satisfy it. On the other hand, TME would take as input only the set S of possible states and the set $\mathcal{E}$ of pieces of evidence. These models make a distinction between evidence and justifications for proposition P, and define a belief operator based on the latter. According to their framework, an argument for P is any non-empty element of the topology generated by $\mathcal{E}$ that is contained in P, while a justification is a subset of P which is dense in that topology. This means that a justification intersects every element of the topology generated by $\mathcal{E}$, so they do not contradict any evidence-based argument for proposition P. Consequently, beliefs based on justifications are very robust even in situations with highly contradictory evidence. In the proposed model we maintain this distinction.

We define our model in three layers. In the first (or qualitative) layer, the agent who receives the input sets a frame of justification. From all the possible combinations of evidence, the agent determines which ones are good enough to justify believing in a proposition that contains it. In this way, the agent is setting its evidential demand. In the second (or quantitative) layer, the elements of $\mathcal{E}^Q$ are combined by computing the mass function over the power set of $\mathcal{E}$ defined in (1). Intuitively, this function distributes the degree of certainty of a piece of evidence among all its possible occurrences in the presence of other pieces of evidence. Finally, the third (or bridging) layer enables the agent to choose a function mapping $2^{\mathcal{E}}$ to the set of justifications. Therefore, the values from the previous layer can be naturally split among the justifications of the first one. This belief model can reproduce both original approaches when appropriate constraints are imposed and it can be used to compute an agent's (possibly) distinct degrees of belief, based on the same evidence, in different situations (for example, when the agent prioritizes avoiding false negatives or false positives).

$$\delta(\boldsymbol{E}) = \prod_{E_i \in \boldsymbol{E}} p_i \prod_{E_i \notin \boldsymbol{E}} (1 - p_i), \text{ where } \boldsymbol{E} \in 2^{\mathcal{E}}. \tag{1}$$

Our next step is to test the suitability of this method in real applications, so we are seeking collaborations with researchers who have experience in handling the problem of combining uncertain evidence. During the poster session, we will present the main technical details of the method, a practical example that shows its full potential, and provide links to the sources with the necessary information to apply this method.

References

[1] Alexandru Baltag, Nick Bezhanishvili, Aybüke Özgün, and Sonja Smets. Justified belief, knowledge, and the topology of evidence. *Synthese*, 200(6):1–51, 2022. doi:10.1007/s11229-022-03967-6.

[2] Glenn Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, Princeton, NJ, 1976.

* *Acknowledgments* We would like to thank the members of the Heudiasyc Laboratory at the University of Technology of Compiègne, especially Sébastien Destercke, Xu Philippe, and Thierry Denœux, for their insightful comments and inputs on Dempster-Shafer Theory and its applications.