Optimization Problems with Evidential Linear Objective*

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This work is centered around an optimization problem with a linear objective (LOP):

$$\begin{aligned} &\max/\min\ c^T x \\ &\quad \text{s.t. } x \in \mathcal{X} \subseteq \mathbb{Z}^{n_1}_{\geq 0} \times \mathbb{R}^{n_2}_{\geq 0} \text{ with } n_1 + n_2 = n. \end{aligned} \tag{LOP}$$

We investigate the case where the uncertainty on the coefficients c_i of c is *evidential*, *i.e.*, modelled by a belief function [2], and where the focal sets of this belief function are Cartesian product of compact sets, with each compact set describing possible values of each coefficient. Such a belief function is a direct and natural generalization of the interval representation found in robust optimization and can be illustrated as follows. A manufacturing company produces two products, A and B. The profit of each product depends on a certain government policy. If the policy is enacted, the profit of product A is either 1000 or 2000 euros, and the profit of product B lies between 1500 and 2000 euros. Otherwise, the profit of product A is either 2000 or 3000 euros, and the profit of product B lies between 1000 and 1500 euros. Additionally, sources predict that the probability of the policy being implemented is 0.6.

We consider five common criteria to compare solutions in this setting: generalized Hurwicz, strong dominance, weak dominance, maximality and E-admissibility [1]. We provide characterizations for the non-dominated solutions with respect to these criteria. These characterizations correspond to established concepts in optimization. More precisely, we show that:

- For any LOP,
  - Non-dominated solutions with respect to the generalized Hurwicz criterion for various α are characterized in terms of solutions of a *parametric* LOP.
  - Non-dominated solutions with respect to strong dominance are characterized in terms of solutions of a *lower-bound feasibility problem*.
  - Non-dominated solutions with respect to weak dominance are characterized in terms of *efficient* solutions of a bi-objective LOP.
  - We provide a sufficient condition for non-dominated solutions with respect to maximality and a necessary condition for non-dominated solutions with respect to E-admissibility both in terms of *possibly optimal solutions* of the LOP.
- When the LOP is a convex optimization problem (*i.e.,* $\mathcal{X}$ is convex) or a combinatorial optimization problem (*i.e.,* $\mathcal{X} \subseteq \{0,1\}^n$), non-dominated solutions with respect to maximality and E-admissibility coincide and are characterized in terms of *possibly optimal solutions* of the LOP. When the LOP is a linear mixed-integer programming (*i.e.,* $\mathcal{X}$ is in the form of $Mx \leq b$ for a matrix M and a vector b), non-dominated solutions with respect to E-admissibility are also characterized in terms of *possibly optimal solutions* of the LOP.

These characterizations make it possible to find non-dominated solutions by solving known variants of the deterministic version of the LOP or even, in some cases, simply by solving the deterministic version. Additionally, in the case of combinatorial optimization problems, our result for E-admissibility is particularly valuable: we show that if the deterministic problem can be solved efficiently (*e.g.,* the shortest path problem), checking E-admissibility is also efficient.

References

[1] Thierry Denoeux. Decision-making with belief functions: a review. *Int. J. Approx. Reason.*, 109:87–110, 2019.

[2] Glenn Shafer. *A mathematical theory of evidence*. Princeton university press, 1976.

*This is a one-page summary of a manuscript with the same title, under review at International Journal of Approximate Reasoning.