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# Representing Suppositional Decision Theories with Sets of Desirable Gambles

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# 3 goals for this talk

- Motivate caring about act-state dependence.
- Briefly introduce Suppositional Decision Theories.
- Get us thinking about what a representation of SDTs with sets of desirable gambles might look like.

# Motivating example: Extortion

Suppose you have just parked in a seedy neighborhood when a man approaches and offers to “protect” your car from harm for $10. You recognize this as extortion and have heard that people who refuse “protection” invariably return to find their windshields smashed. Those who pay find their cars intact. You cannot park anywhere else because you are late for an important meeting. It costs $400 to replace a windshield. Should you buy “protection”? (James Joyce, *Foundations of Causal Decision Theory,* p. 115)

# Dominance reasoning fails

| | Broken | Unbroken |
|---|---|---|
| Pay | -$410 | -$10 |
| Don't Pay | -$400 | 0 |

- It seems like there's a *dominance* argument against Paying: either your windshield will be Broken or Unbroken.
- In either case, Don't Pay provides strictly more value (you keep $10 more) than Pay.
- But… this is silly.

# Diagnosis

- Obvious problem: whether your windshield breaks *depends* on whether you pay.
- Static belief model on the states is obviously insufficient; we need a model that captures this dependence.
- But what do we mean by “depends”?
- Many different things: e.g., we might mean epistemic dependence or causal dependence; and there are many ways we might analyze even just these.

# Suppositional Decision Theories

- Well-studied in philosophy, but almost exclusively in a precise context.
- Basic idea: evaluate each act $A$ from the epistemic perspective of "supposing" $A$ is the act you perform.
- Supposition rule: $s: \mathbb{P} \times \mathcal{P}_{\neg\emptyset}(\Omega) \to \mathbb{P}$; maps prior probability and supposed event to a new probability function.
- Supposing an event requires certainty: $s(p, R)(\omega) = 0$ for any $\omega \notin R$.
- Pick the act which maximizes suppositional expected utility:

$$V_s(p, u, \cdot) = E_{s(p,\cdot)}(u) = \sum_{\omega \in \Omega} s(p, \cdot)(\omega) u(\omega).$$

# Suppositional Decision Theories

- Different supposition rules can represent a wide array of different policies concerning how supposing an event impacts other beliefs.
- E.g., Bayesian conditionalization as the supposition rule yields Richard Jeffrey's Evidential Decision Theory.
- Can also represent various Causal Decision Theories (e.g., supposition as Pearl's do-operator).
- Can even represent some more exotic decision theories, like Functional Decision Theory.
- Typically, CDTs have a special representation in terms of *generalized imaging.* (What's that? Ask me at the poster.)

# Obvious ways to add imprecision to SDTs

- Rather than assuming the agent has a precise prior probability function, allow an imprecise prior represented by a set of probability functions, $P \subseteq \mathbb{P}$.
- One natural decision rule would be

  $A \succ B$ iff $(\forall p \in P)\big(V_s(p, u, A) > V_s(p, u, B)\big)$.
- We could also allow for imprecision about the supposition rule itself (see the paper for more on this).

# Comparison case: SDG representation in the act-state *independent* context

- In the case where acts and states are *independent*, it is sufficient to represent the agent's beliefs about states.
- So, assume an imprecise prior $P \subseteq \mathbb{P}(\mathcal{X})$.
- Agent has a utility function on the total outcome space $\Omega \subseteq \mathcal{A} \times \mathcal{X}$; $u: \Omega \to \mathbb{R}$, reflecting all preferences relevant to the decision.
- Suppose agent's preferences are determined by "supervaluation":

$$A \succ B \text{ iff } (\forall p \in P)\left(E_p(u_A) > E_p(u_B)\right),$$

with $E_p(u_A) = \sum_{X \in \mathcal{X}} p(X) u(A, X)$.

# SDGs in the *independent* context

- This has a nice representation in terms of sets of desirable gambles.
- A gamble $g: \mathcal{X} \to \mathbb{R}$ represents a gain/loss to the agent determined by which *state* obtains.
- $\mathcal{L}(\mathcal{X})$ is the set of all such gambles.
- A gamble is *desirable* iff $g \succ 0$; the 0 gamble represents the status quo.
- $D \subseteq \mathcal{L}(\mathcal{X})$ is the agent's set of desirable gambles.

Then, the agent's preferences over acts can be represented by an SDG $D$ as follows:

1. Each act $A \in \mathcal{A}$ has a characteristic gamble $g_A(X) = u(A, X)$.
2. Link between $D$ and $P$: $g \in D$ iff $(\forall p \in P)\big(E_p(g) > 0\big)$.
3. Read act preferences from gamble desirability: $A \succ B$ iff $g_A - g_B \in D$.
4. Act pricing: agent will pay $\epsilon$ for $A$ iff $g_A - \epsilon \in D$; agent will sell $A$ for $\epsilon$ iff $\epsilon - g_A \in D$.

Can we use SDGs to model supervaluated SDTs in a similar way?

Come see
my poster!

# References (selected)


Andrew Bacon. Actual value in decision theory. *Analysis*, forthcoming. doi: 10.1093/analys/anac014.

Peter Gärdenfors. Imaging and conditionalization. *The Journal of Philosophy*, 79(12):747–760, 1982. ISSN 0022362X. URL http://www.jstor.org/stable/2026039.

James M. Joyce. *The Foundations of Causal Decision Theory*. Cambridge Studies in Probability, Induction and Decision Theory. Cambridge University Press, 1999. doi: 10.1017/CBO9780511498497.

Judea Pearl. *Causality: Models, Reasoning and Inference (2nd ed.)*. Cambridge University Press, 2009.

Eliezer Yudkowsky and Nate Soares. Functional decision theory: A new theory of instrumental rationality, 2017. URL https://arxiv.org/abs/1710.05060.

Jiji Zhang, Teddy Seidenfeld, and Hailin Liu. Subjec- tive causal networks and indeterminate suppositional credences. *Synthese*, 198(Suppl 27):6571–6597, 2019. doi: 10.1007/s11229-019-02512-2.