

Vertical Barrier Models as Unified Distortions

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What is a Vertical Barrier Model (VBM)?

- ▶ A VBM is a *distortion model*: given a probability P_0 on $\mathcal{P}(\Omega)$, it obtains a lower probability \underline{P} and its conjugate \overline{P} by:

$$\underline{P}(A) = \max\{bP_0(A) + a, 0\}, \forall A \neq \Omega, \underline{P}(\Omega) = 1$$

when $a \leq 0, b > 0, a + b \in [0, 1]$

$$\overline{P}(A) = \min\{bP_0(A) + c, 0\}, \forall A \neq \emptyset, \overline{P}(\emptyset) = 0, c = 1 - (a + b)$$

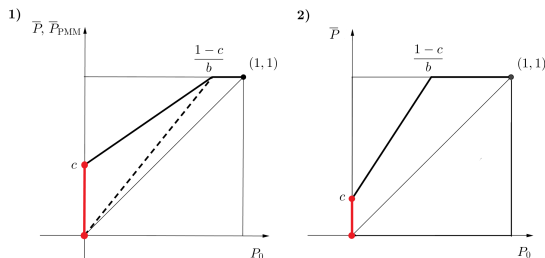
- ▶ VBMs belong to the family of *Nearly-Linear Models* (NLM), obtained varying the constraints on a, b .

NLMs are often only 2-coherent, while for VBMs

\underline{P} is coherent and 2-monotone.

Reasons for Employing VBMs

- ▶ They incorporate other well known models: the **Pari-Mutuel Model (PMM)**, **Total Variation Model (TV)**, **Linear-Vacuuous Mixture (LV)**.



- ▶ Behaviourally, the selling price $\bar{P}(A)$ ensures a **loading** $\bar{P}(A) - P_0(A) \geq 0$ over the 'fair price' $P_0(A)$.
VBMs allow a variety of loading policies.
- ▶ Computations with VBMs are (relatively) simple.

Properties of VBMs

- ▶ Properties of VBMs established by Corsato, Pelessoni, Vicig (2020, 2021) and Pelessoni, Vicig (2022) include:
 - Formulae for the natural extension.
 - Conditioning with VBMs.
 - Dilation of VBMs.
 - Characterisation of VBMs that are probability intervals.
- ▶ Related investigations by Destercke, Miranda, Montes on the PMM (2019) and neighbourhood/distortion models (2020a, 2020b) considered (also) complementary issues.
- ▶ This paper explores further aspects of VBM, building upon these previous results.

Results in this Paper

- ▶ VBMs as neighbourhood models.
- ▶ Structure of the credal set of \underline{P} in a VBM.
- ▶ Connection with other distortion models.
- ▶ Processing VBMs: conjunction, disjunction, mixture of two VBMs.
- ▶ VBMs that are also belief functions and possibility measures.

In the paper, the possibility space Ω is finite, while neither \underline{P} nor P_0 are necessarily strictly positive.

VBM as Neighbourhood Models

- ▶ Let $\mathcal{M}(\underline{P}) = \{P \text{ probability} \mid P(A) \geq \underline{P}(A), \forall A\}$ be the **credal set** associated with \underline{P} .
- ▶ Given a VBM (P_0, a, b) , with $a < 0 < b$, $a + b < 1$, define the premetric/distorting function

$$d_{\text{VBM}} = \max_{A \subset \Omega} \frac{P_0(A) - P(A)}{(1-b)P_0(A) - a}$$

- ▶ Then, the VBM (P_0, a, b) can be seen as the neighbourhood model determined by d_{VBM} around P_0 , meaning that

$$\mathcal{M}(\underline{P}) = \{P \text{ probability} \mid d_{\text{VBM}}(P, P_0) \leq 1\}$$

- ▶ The definition of d_{VBM} can be extended to the case of $a = 0$ or $a + b = 1$ and then it gives the distorting functions of the PMM or the LV models.

Credal Set of a VBM

- ▶ Given a VBM \underline{P} on a space Ω of cardinality n , the maximum number of extreme points of $\mathcal{M}(\underline{P})$ is

$$\frac{n!}{\lfloor \frac{n}{2} - 1 \rfloor! \lceil \frac{n}{2} - 1 \rceil!}$$

- ▶ When in particular $\underline{P}(\{\omega\}) > 0, \forall \omega \in \Omega$, the maximum number of extreme points of $\mathcal{M}(\underline{P})$ is $n(n-1)$.
- ▶ The bounds are the same as for TV models, but greater than for PMM or LV models.

VBMs and Belief Functions

Let \underline{P} be a VBM.

- ▶ If $|\Omega| = 3$ and $b \leq 1$ then \underline{P} is a belief function.
- ▶ If \underline{P} is a belief function, then there cannot exist $A_1, A_2, A_3 \in \mathcal{P}(\Omega)$ such that $A_i \cap A_j = \emptyset \forall i \neq j$, $\underline{P}(A_i) > 0 \forall i$ and $A_1 \cup A_2 \cup A_3 \neq \Omega$.

Thus, if $|\Omega|$ is large enough, a VBM will have to assign lower probability zero to the majority of events, and hence to be almost vacuous, for \underline{P} to be a belief function.

Other necessary and sufficient conditions for \underline{P} to be a belief function at the poster session.

Conclusions

- ▶ Although they do not reduce to preexisting models in general, VBMs are not much more complex than their special cases, as shown by their distorting function and by the structure of their credal sets.
- ▶ Further work on: structure of the mass function of a VBM, VBMs in approximation problems for coherent upper/lower probabilities.

For details, come and meet us at the poster session!



Enrique and Paolo
(in a hard work session)



Renato
(the one at the wheel)