Robust possibilistic production planning under temporal demand uncertainty with knowledge on dependencies

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Outline of the presentation

Context

Production planning

The production planning is composed of three levels of decisions:

- the strategical level:
  - long horizon (e.g. 5 year)
  - large discretisation (e.g. 1 month)
  - decisions taken with a low frequency (e.g. capacity decisions)
- the tactical level:
  - middle horizon (e.g.1 years)
  - middle discretisation (e.g.1 week)
  - decisions taken to anticipate the production quantity
- the operational level
  - sort horizon (e.g.1 week)
  - small discretisation (e.g. 1 hour)
  - precise decision of production

Motivations and solution approach

- Knowledge on demand is uncertain due to fast market evolution.
- Uncertainty over the date on which demand will be requested impacts production planning
- Decision-makers have a good knowledge of possible changes and modify demand forecast according to their beliefs before computing the optimal solution

Approach proposed

The decision-maker defined him knowledge on possible set of demands form the forecast and then optimize taking into account the imprecise knowledge.

Formulation of Production Planning Problem

Our problem can be modeled by the following linear program:

$$
\begin{aligned}
\min \sum_{t\in[T]} & (c^I I_t + c^B B_t + c^P x_t - b^P s_t) \\
\text{s.t. } & B_t - I_t = D_t - X_t && t \in [T], \\
& \sum_{i\in[t]} s_i = D_t - B_t && t \in [T], \\
& B_t, I_t, s_t \geq 0, \boldsymbol{x} \in \mathbb{X} \subseteq_+^T && t \in [T]
\end{aligned}
$$

where $D_t = \sum_{i\in[t]} d_i$ and $X_t = \sum_{i\in[t]} x_i$, D_t and X_t stand for the cumulative demand up to period t and the cumulative production up to period t, respectively. I_t and B_t are respectively the inventory and the backordering à period t

Temporal demand uncertainty

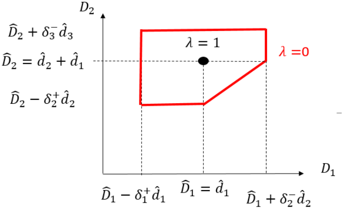
let $\hat{d}_t$ be the forecasting demand at period t and $\hat{D}_t = \sum_{i \in [t]} \hat{d}_i$ the cumulative forecasting demand.

imprecision on the realization of the demand

The manager can give for each period $t \in [T]$

- the maximum percent of demand $\delta_t^- \in [0, 1]$ that can be advanced to the previous period $t - 1$ and
- the maximum percent of demand $\delta_t^+ \in [0, 1]$ that can be delayed to the next period $t + 1$.
- the level of dependency between horizon periods

$$
\begin{aligned}
\mathrm{D}^{[\lambda]} : \quad & \{\boldsymbol{D}^{[\boldsymbol{\lambda}]} \in^T \\
s.t. \quad & D_t^{[\lambda]} \geq \hat{D}_t - \beta_t^+ \delta_t^+ \hat{d}_t, \forall t \in [T] \\
& D_t^{[\lambda]} \leq \hat{D}_t + \beta_{t+1}^- \delta_{t+1}^- \hat{d}_{t+1}, \forall t \in [T \\
& \beta_t^- \leq 1 - \lambda, \forall t \in [T] \\
& \beta_t^+ \leq 1 - \lambda, \forall t \in [T] \\
& \beta_t^+ \delta_t^+ + \beta_t^- \delta_t^- \leq 1, \forall t \in [T] \\
& \beta_t^-, \beta_t^+ \in [0, 1], \forall t \in [T]\}
\end{aligned}
$$



Π−RO

Robust possibilistic optimization maximize the degree of certainty that a solution cost is lower than a given threshold g.

$$\max_{\boldsymbol{x}\in\mathbb{X}} \quad N(\mathcal{C} \leq g)$$

The necessity measure is computed from the marginal possibility distribution and a function $H_\alpha : [0,1]^T \to [0,1]$ (eq.1) where $\alpha \in [0,1]$ is the degree of belief that they is negative dependencies, i.e.

- if $\alpha = 0$ we do not have negative dependencies between uncertainty,
- if $\alpha = 1$ we know that there is negative dependencies between uncertainty of each periods.

$$H_\alpha(\boldsymbol{u}) = \alpha.W(\boldsymbol{u}) + (1-\alpha)M(\boldsymbol{u}), \alpha \in [0,1] \qquad (1)$$

with $W(\boldsymbol{u}) = \max\{\sum_{i=1}^{T} u_i - T + 1, 0\}$ and $M(\boldsymbol{u}) = \min_{i=1,\dots,T}(u_i)$.

The necessity degree to have a vector $\boldsymbol{x} \in^T$ lower or equal to a given vector $\boldsymbol{a}$ is:

$$N(\boldsymbol{x} \leq \boldsymbol{a}) = 1 - H_\alpha(\Pi(a_1 < x_1), \dots, \Pi(a_T < x_T))$$

Problem formulation

The problem with time uncertainty can be formulate as particular case of production planning with uncertain cumulative demands $\widetilde{\boldsymbol{D}} = (\widetilde{D}_t)_{t\in[T]}$:

$$\widetilde{\min_{\boldsymbol{x}\in\mathbb{X}}} \mathcal{C}(\boldsymbol{x}, \widetilde{\boldsymbol{D}}) = \widetilde{\min_{\boldsymbol{x}\in\mathbb{X}}} \sum_{t\in[T]} \mathcal{C}_t(x_t, \widetilde{D}_t) = \tag{2}$$

$$\sum_{t\in[T]} \max\{c^I(X_t - \widetilde{D}_t), c^B(\widetilde{D}_t - X_t)\} + c^P X_T$$

$$-b^P \min\{X_T, \widetilde{D}_T\}. \tag{3}$$

Using $\Pi-$RO:

$$\begin{aligned} \min_{\boldsymbol{x}\in\mathbb{X}} \quad & \epsilon \\ & \max_{\{\boldsymbol{c}|H_\alpha(\boldsymbol{c})\geq\epsilon\}} \sum\nolimits_{t\in[T]} c_t \leq g \\ & \mathcal{C}_t = c_t \qquad \forall t \in [T] \end{aligned} \tag{4}$$

with $H_\alpha(\boldsymbol{c}) = H_\alpha(\Pi(c_1 < C_1), ..., \Pi(c_T < C_T))$

Solving the adversarial problem

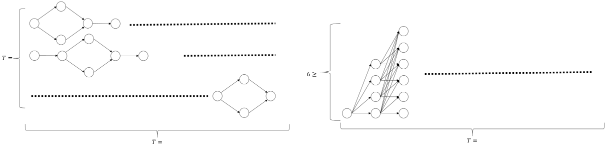
First we focus on the problem :

$$\max_{\{\mathcal{C}=\boldsymbol{c} \mid H_\alpha(\boldsymbol{c}) \geq \epsilon\}} \quad \begin{array}{l} \sum_{t\in[T]} c_t \\ \boldsymbol{D} \in \boldsymbol{D}^{[0]} \end{array} \tag{5}$$

Theorem

The problem 5 can be computed in $O(T^2)$ times

The main idea of the proof is that the problem can be formulated as longest path problem for graphs with one of the following structures :



Solving Problem

for a given ϵ

Using the dual formulation of the longest path problem formulation of the adversarial problem we can represent the problem by a linear programme.

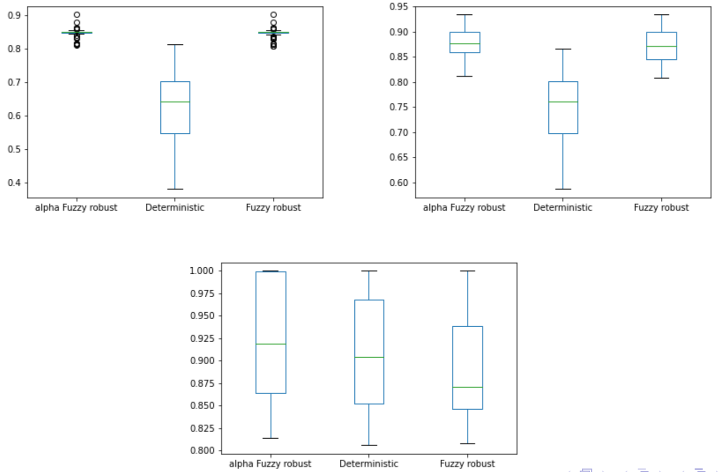
minimize ϵ

$\epsilon \in [0, 1]$ due to the fact that h^{ϵ} is nonincreasing function of ϵ. We can apply binary search on $[0, 1]$ and at each iteration if they exist a feasible solution which boils down to solving the solve the linear programming formulation of the problem above.

The running time of the above algorithm is $O(I(T) \log \xi^{-1})$ time, where $I(T)$ is the time required for solving the linear program, $\xi > 0$ is the error tolerance.

Experimental results

Degree of necessity of different types of optimal solutions for $\alpha = 0.25$, $\alpha = 0.5$ and $\alpha = 1$



See you at the poster session!!