The logic FP(Ł, Ł) and two-sorted equational states

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The logic $FP(\text{Ł},\text{Ł})$ and two-sorted equational states

The logic FP(Ł, Ł) and

two-sorted equational states

Let us start with a simpler logic

$$FP(Ł, Ł)$$

Fuzzy

Probability

Łukasiewicz logic

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Łukasiewicz logic

Recall that Łukasiewicz logic is a many-valued logic with truth values ranging in $[0,1]$. Its algebraic semantics is given by MV-algebras

$$(A, \oplus, \neg, 0)$$

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Fuzzy + Modality = Probability

$\Box \varphi$ to be read as "φ is probable"

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Boolean formula

This is a classical event!

Fuzzyness is only used to speak about degrees of probability

Let us now get to our system

$$\underbrace{F}_{\text{Fuzzy}}\ \underbrace{P}_{\text{Probability}}(\underbrace{\text{Ł}, \text{Ł}}_{\text{Łukasiewicz logic}})$$

Let us now get to our system

$$FP(\underbrace{Ł,Ł}_{\text{Łukasiewicz logic}})$$

Fuzzy Probability

Fuzzy + Modality = Probability

$\Box\varphi$ to be read as "φ is probable"

Łukasiewicz formula

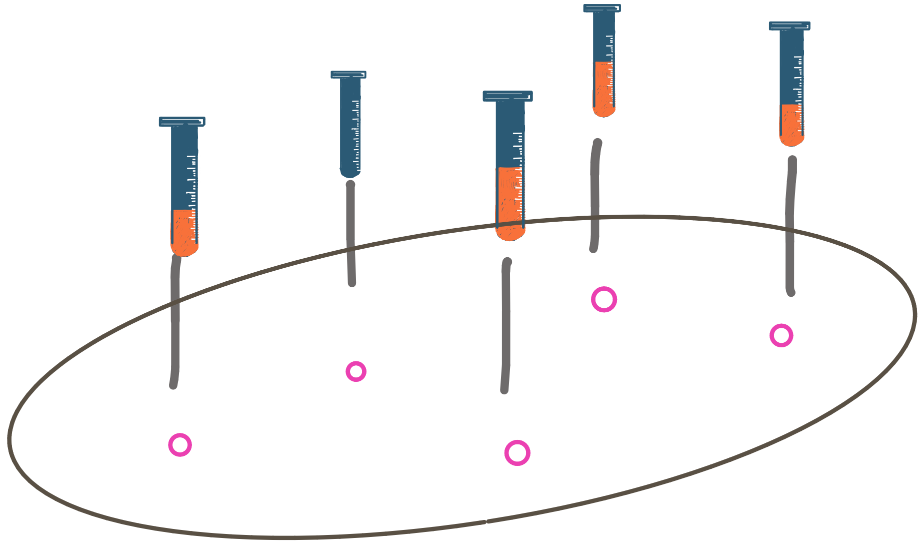
This is now a fuzzy event!

FP(Ł) was introduced in 1995 by Esteva, Hájek and Godo to formally reason about probabilities on classical events.

Theorem (1995) The logic FP(Ł) is complete w.r.t. probability functions over Boolean algebras

FP(Ł, Ł) was introduced in 2007 by Flaminio and Godo to reason about probabilities on vague events.

Theorem (Flaminio 2021) The logic FP(Ł, Ł) is complete w.r.t. state models



$\xrightarrow{s} [0,1]$

The logic $FP(\text{Ł}, \text{Ł})$ and

two-sorted equational states

A state (on a lattice ordered group) is a positive, normalized, group homomorphism into $\mathbb{R}$

$$s : G \longrightarrow \mathbb{R}$$

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$\mathbb{R}$ has itself a structure of lattice ordered group!

$$s : G \longrightarrow G'$$

A two-sorted algebra is given by two sets A_1, A_2 and typed operations.

E.g. $*: A_1 \times A_1 \to A_1$
$+: A_2 \times A_2 \to A_2$
$\cdot: A_1 \times A_2 \to A_2$
$\vdots$

Example Graphs can be presented as two-sorted algebras:

$$(N, E, \mathrm{dom}: E \to N, \mathrm{cod}: E \to N)$$

With a bit of care all constructions in general algebra work well in multi-sorted algebras!

[Kroupa - Marra 2021]

Two - sorted equational states are two-sorted algebras (A_1, A_2) in which :

1) Each sort A_i $i = 1,2$ is equipped with the structure of an MV - algebra.

2) There is a further operation $s : A_1 \to A_2$ subject to the laws:

a) $s(1) = 1$
b) $s(\neg a) = \neg s(a)$
c) $s(a \oplus b) = s(a) \oplus s(b \wedge \neg a)$

Two-sorted equational states are called so because they are axiomatized by (two-sorted) equations.

Thus, many tools and concept of general algebra can be employed in the study of equational states:

1) Free equational states
2) Generic equational states
3) Homomorphisms of equational states

⋮

Recap

$FP(Ł, Ł)$

logical system to reason about probability of vague events

Two-sorted equational states

Equational algebraic presentation of probability on classical events

Our result

The logic $FP(Ł, Ł)$ and two-sorted equational states

Our result

Theorem: The logic $FP(\L,\L)$ is complete with regards to the class of two-sorted equational states. In other words:

$$\boxed{FP(\L,\L) \vdash \varphi \iff ES \models \varphi^{=} = 1}$$

1. How do we prove this?
2. What are the consequences?
3. What applications does this provide?
4. What are the next questions to be addressed?

Want to know more?

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Want to know more?

Come to the poster!

Thank you