Los Alamos NATIONAL LABORATORY

Open World Dempster-Shafer using Complementary Sets

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Overview

Dempster-Shafer Theory (DST) is a mathematical framework for reasoning with uncertainty.

DST

- Assumes a closed-world
- Conflict is conflated with unknown propositions

Complementary DST (CDST)

- Uses complements of sets to achieve an open-world
- Distinguishes between conflict and the unknown
- Apply to land cover classification problem

DST: The Closed World

- A *Frame of Discernment*, Ω, is a collection of propositions or hypothesis.
- *Focal Elements* are members of the powerset of the frame of discernment

$$u \in 2^{\Omega} .$$

  - The closed world.

Example

Frame of Discernment: $\Omega = \{A, B, C\}$

Focal Elements: $2^{\Omega} =$

$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{A, B\}$	$\{A, C\}$	$\{B, C\}$	$\{A, B, C\}$

CDST: The Open World

- *Complementary Focal Elements*: members of the Cartesian product between the power set of a frame of discernment and the Boolean space,

$$(u, a) \in 2^{\Omega} \times \mathbb{B} \ .$$

  - (u, T) represents the hypothesis of u
  - (u, F) represents the hypothesis of everything other than u, including all propositions not in Ω

Example

Frame of Discernment : $\Omega = \{A, B, C\}$

Complementary Focal Elements : $2^{\Omega} \times \mathbb{B} =$

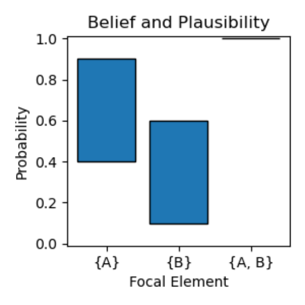
$(\emptyset, T)$	$(\{A\}, T)$	$(\{B\}, T)$	$(\{C\}, T)$	$(\{A, B\}, T)$	$(\{A, C\}, T)$	$(\{B, C\}, T)$	$(\{A, B, C\}, T)$
$(\{A, B, C\}, F)$	$(\{B, C\}, F)$	$(\{A, C\}, F)$	$(\{A, B\}, F)$	$(\{C\}, F)$	$(\{B\}, F)$	$(\{A\}, F)$	$(\emptyset, F)$

DST: Components

- *Basic Probability Assignments* (BPA): real nonnegative function on focal elements that sums to one
  - In DST $m(\emptyset) = 0$, but not in Transferable Belief Models or Generalized Evidence Theory.
- Belief: lower probability bound of BPA
  - $\text{Belief}_m(x) = \sum_{y \subseteq x} m(y)$
- Plausibility: upper probability bound of BPA
  - $\text{Plausibility}_m(x) = \sum_{y \cap x \neq \emptyset} m(y)$
- Rules Of Combination: Mechanisms to fuse together BPAs

Example

$\Omega = \{A, B\}$
$m = \{\{A\} : 0.4, \{B\} : 0.1, \{A, B\} : 0.5\}$

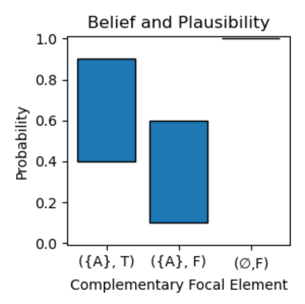


CDST: Components

- *Complementary Basic Probability Assignments* (CBPA): real nonnegative function on complementary focal elements that sums to one
- Belief: lower probability bound of CBPA
  - $\text{Belief}_m(x) = \sum_{y \subseteq x} m(y)$
- Plausibility: upper probability bound of CBPA
  - $\text{Plausibility}_m(x) = \sum_{y \cap x \neq \emptyset} m(y)$
- Rules Of Combination: Mechanisms to fuse together CBPAs

Example

$\Omega = \{A\}$
$m = \{(\{A\}, T) : 0.4, (\{A\}, F) : 0.1, (\emptyset, F) : 0.5\}$



DST: Zadeh's Criticism

Example

Ask your subject matter experts: "How much do you believe it is *meningitis*, *concussion*, and *tumor*?"

- SME1: "It's probably *meningitis*, with a slight chance it's *concussion*."
  - $m_1 = \{\{meningitis\} : 0.99, \{meningitis, concussion\} : 0.01\}$
- SME2: "It's probably *tumor*, with a slight chance it's *concussion*."
  - $m_2 = \{\{tumor\} : 0.99, \{concussion, tumor\} : 0.01\}$

$m_{1,2} = \{\emptyset : 0.9999, \{concussion\} : 0.0001\}$

Fundamental conflict between SMEs, $m_{1,2}(\emptyset) = 0.9999$
Believing $m(\{concussion\}) = 0.0001$ unlikely, but only choice[a]

[a] Example adapted from **[L. A. Zadeh. On the validity of Dempster's rule of combination of evidence. University of California, Berkeley, 1979.]**

CDST: Open World Resolution to Zadeh's Criticism

Example

Ask your subject matter experts: "How much do you believe it is not *meningitis*, *concussion*, and *tumor*?"

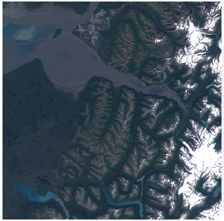
- SME1: "It's definitely not *tumor*, and probably not *concussion*."
  - $m_1 = \{(\{concussion, tumor\}, F) : 0.99, (\{tumor\}, F) : 0.01\}$
- SME2: "It's definitely not *meningitis*, and probably not *concussion*."
  - $m_2 = \{(\{meningitis, concussion\}, F) : 0.99, (\{meningitis\}, F) : 0.01\}$

$m_{1,2} = \{(\{meningitis, concussion, tumor\}, F) : 0.9999, (\{meningitis, tumor\}, F) : 0.0001\}$

No conflict between SMEs, $m((\emptyset, T)) = 0$
Most likely an unconsidered proposition,
$(\{meningitis, concussion, tumor\}, F) : 0.9999$

CDST: Land Cover Classification Application

- Applied to Sentinel-2 Satellite imagery of Anchorage Alaska
- Use normalized difference metrics to identify materials and construct CBPAs
- CBPAs created in ignorance of other land cover classes
- Separates conflict from unknown propositions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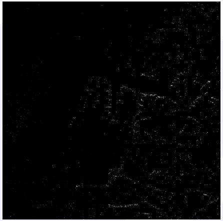
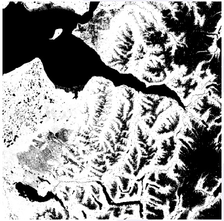

$(\emptyset, T)$


$(\{water\}, T)$


$(\{veg.\}, T)$


$(\{snow\}, T)$


$(\{w., veg., s.\}, F)$

Conclusion

Complementary DST

- Open-world
- Separates conflicting evidence from unconsidered propositions
- Allows for the explicit assignment of mass to ignorance

Backup Slides

Application: Land Cover Classification

SMEs use normalized differences

$$\text{NormalizedDifference}(Band_i, Band_j) = \frac{Band_i - Band_j}{Band_i + Band_j}$$

of multispectral satellite bands to identify specific land cover classes.

- Large values: presence of material
- Small values: absence of material

Normalized difference indices:

- Water Index: assigns $(\{water\}, T)$ and $(\{water\}, F)$
- Vegetation Index: assigns $(\{vegetation\}, T)$ and $(\{vegetation\}, F)$
- Snow Index: assigns $(\{water, snow\}, T)$ and $(\{water, snow\}, F)$