# Constriction for Sets of Probabilities

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# Motivation

- Bayes' rule $P(A \mid E) = \frac{P(E|A)P(A)}{P(E)}$ is arguably the most common updating rule for subjective beliefs
- Its IP version, called generalized Bayes' rule is given by

$$\underline{P}(A \mid E) = \inf_{P \in \mathcal{P}} \frac{P(E \mid A)P(A)}{P(E)} \quad \text{and} \quad \overline{P}(A \mid E) = \sup_{P \in \mathcal{P}} \frac{P(E \mid A)P(A)}{P(E)}$$

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- "When we observe new evidence in the form of a partition $\mathcal{E}$, can we have that $[\underline{P}(A \mid E), \overline{P}(A \mid E)] \subset [\underline{P}(A), \overline{P}(A)]$, for all $E \in \mathcal{E}$?"

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  - Dilation – i.e. the opposite inclusion – is commonplace for IP neighborhood models[1]
  - Constriction is possible only in specialized settings
- "That is odd and fascinating! Can you tell us more about those settings?"
- That's the question we explore in this paper

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- Examples
  - de Finetti's Fundamental Theorem of Probability (de Finetti, 1974, Section 3.10)
  - Halmos' Extension (Halmos, 1950, Exercise 48.4), (Billingsley, 1995, Section 4.13)
  - Jaynes' MaxEnt Kesavan (2008)
  - Generalized Fiducial Inference Hannig et al. (2016)

# Convex pooling

- DeGroot consensus model DeGroot (1974)
- $k$ individuals, each having their own subjective probability distribution
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- $k$ individuals, each having their own subjective probability distribution
- For agent $i$, the opinions of all the other $k-1$ agents represent new evidence
- Instead of conditioning on those, agent $i$ pools their own opinion with that of the other agents
- Repeating this process for all agent $i$, the group reaches (asymptotically) an agreement on a common subjective probability distribution

# Non-Bayesian updating

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  - Intentional forgetting Golding and MacLeod (1998)

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  - Dempster's updating rule Dempster (1967)
  - Gärdenfors' generalized imaging updating rule Gärdenfors (1988)

# Carthago delenda est

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# References I

Patrick Billingsley. *Probability and Measure*. New York, NY : Wiley, second edition, 1995.

Bruno de Finetti. *Theory of Probability*, volume 1. New York : Wiley, 1974.

Morris H. DeGroot. Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121, 1974.

Arthur Pentland Dempster. Upper and lower probabilities induced by a multivalued mapping. *The Annals of Mathematical Statistics*, 38(2): 325–339, 1967.

Jonathan M. Golding and Colin M. MacLeod. *Intentional forgetting: Interdisciplinary approaches*. Mahwah, NJ : Lawrence Erlbaum Associates, 1998.

Ruobin Gong and Xiao-Li Meng. Judicious judgment meets unsettling updating: dilation, sure loss, and Simpson's paradox. *Statistical Science*, 36(2):169–190, 2021.

# References II

Peter Gärdenfors. *Knowledge in flux: Modeling the dynamics of epistemic states*. Boston, MA : The MIT press, 1988.

Paul R. Halmos. *Measure Theory*. Graduate Texts in Mathematics. New York, NY : Springer, 1950.

Jan Hannig, Hari Iyer, Randy C. S. Lai, and Thomas C. M. Lee. Generalized fiducial inference: A review and new results. *Journal of the American Statistical Association*, 111(515):1346–1361, 2016.

Hiremagalur K. Kesavan. Jaynes' maximum entropy principle. In Christodoulos A. Floudas and Panos M. Pardalos, editors, *Encyclopedia of Optimization*. Boston, MA : Springer, 2008.

Isaac Levi. Why indeterminate probability is rational. *Journal of Applied Logic*, 7(4):364–376, 2009.