Uncertainty Propagation using Copulas in a 3D Stereo Matching Pipeline

Roman Malinowski   Sébastien Destercke   Emmanuel Dubois
Loïc Dumas   Emmanuelle Sarrazin

CNES - CS Group - Université de Technologie de Compiègne

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CNES CO3D mission

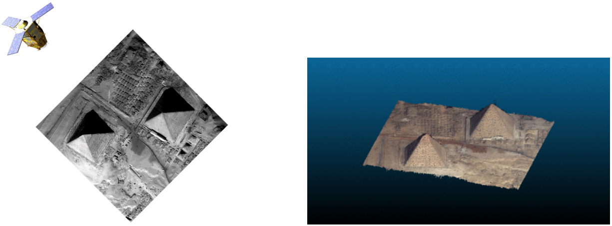

Figure: CARS 3D pipeline [Youssefi et al., 2020]

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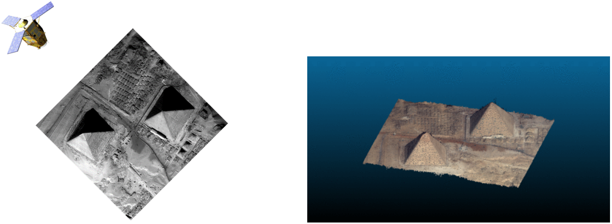
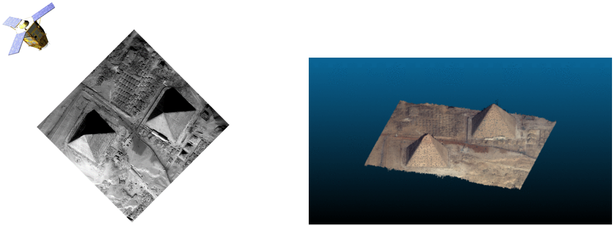


Figure: CARS 3D pipeline [Youssefi et al., 2020]

- Uncertainty on the input images

CNES CO3D mission



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- Uncertainty on the input images
- Caracterizing the uncertainty of the output height map

Stereo-matching

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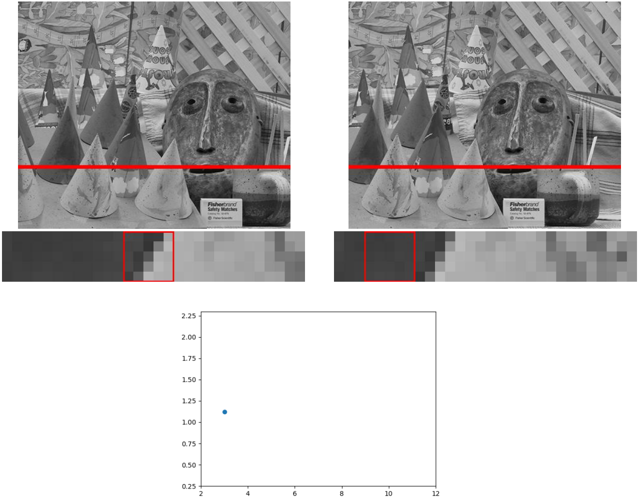
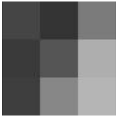
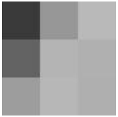


Figure: Cost function [Cournet et al., 2020]

Cost between two patches





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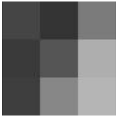

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58	85	174
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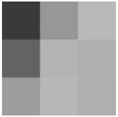

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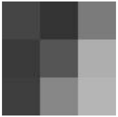

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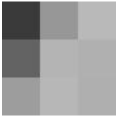

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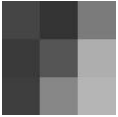

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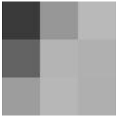

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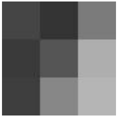

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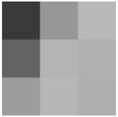

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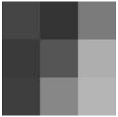

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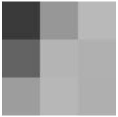

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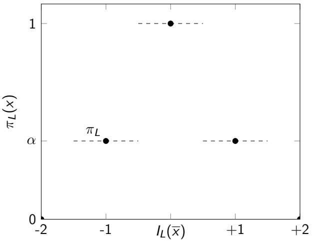
We use possibility distributions as our uncertainty models



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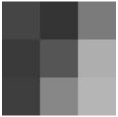

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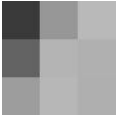

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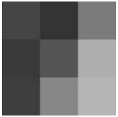

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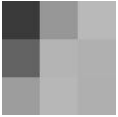

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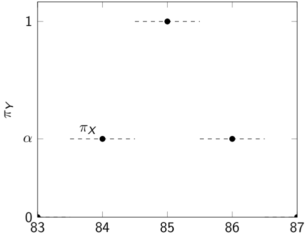

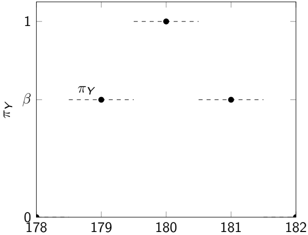

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We use copulas as aggregation models, using Sklar's theorem

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Theorem ([Sklar, 1959])

Every multivariate CDF F can be expressed in terms of its marginals $F_i(x_i) = P(X_i \leqslant x_i)$ and a unique copula C:
$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$

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Ideally, we would like to know

$$\mathcal{M}_{robust} = \{P_{XY} \mid F_{XY} = C(F_X, F_Y)\}$$

with $F_X \in \mathcal{M}(\pi_X), F_Y \in \mathcal{M}(\pi_Y)$

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This set is hard to compute.

Applying copulas to belief functions

Another approach is to apply the copula to the mass functions [Ferson et al., 2004]:

$$m_{XY}(A,B) = \Delta C(\sum_{a \leqslant A} m_X(a), \sum_{b \leqslant B} m_Y(b))$$

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Can $\mathcal{M}_{mass}$ be an (outer) approximation of $\mathcal{M}_{robust}$?

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Useful to boost computation

Monte Carlo

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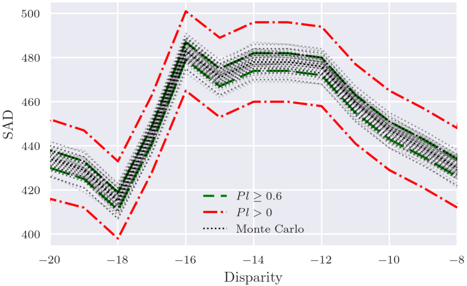

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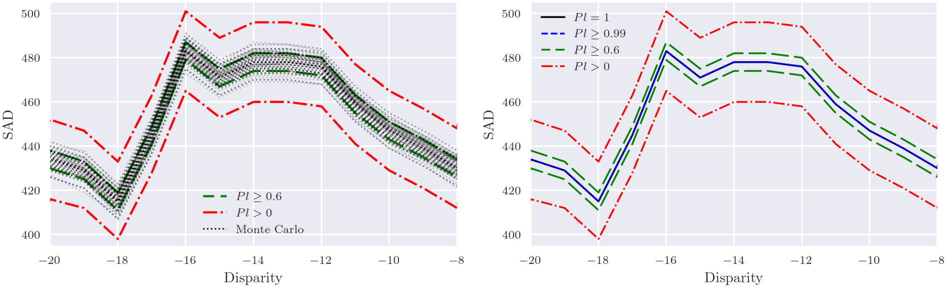

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