# Markov Conditions and Factorization in Logical Credal Networks

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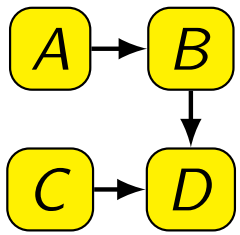
ISIPTA 2023

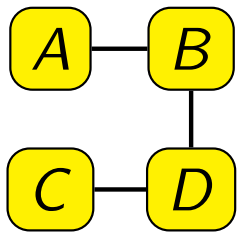
# Overview:

- Paper examines the recently proposed language of *Logical Credal Networks*.
- Focus: connection between Markov conditions and factorization.
- For networks where *structure* is a chain graph, factorization is obtained.

# Graphs, Markov conditions, factorizations...



(DAG)



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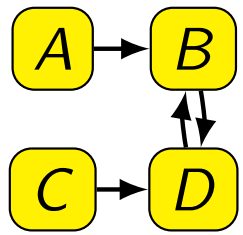

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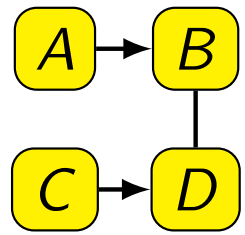

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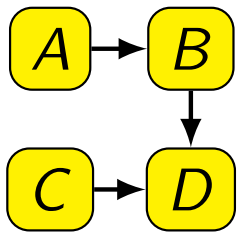
# Graphs, Markov conditions, factorizations...



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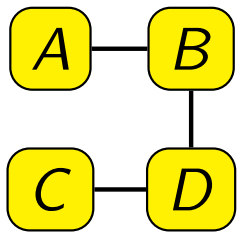

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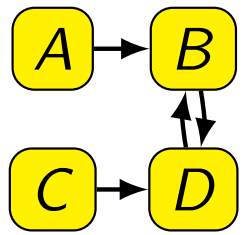

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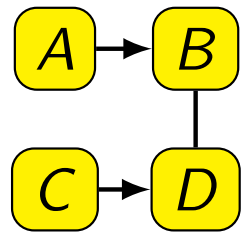


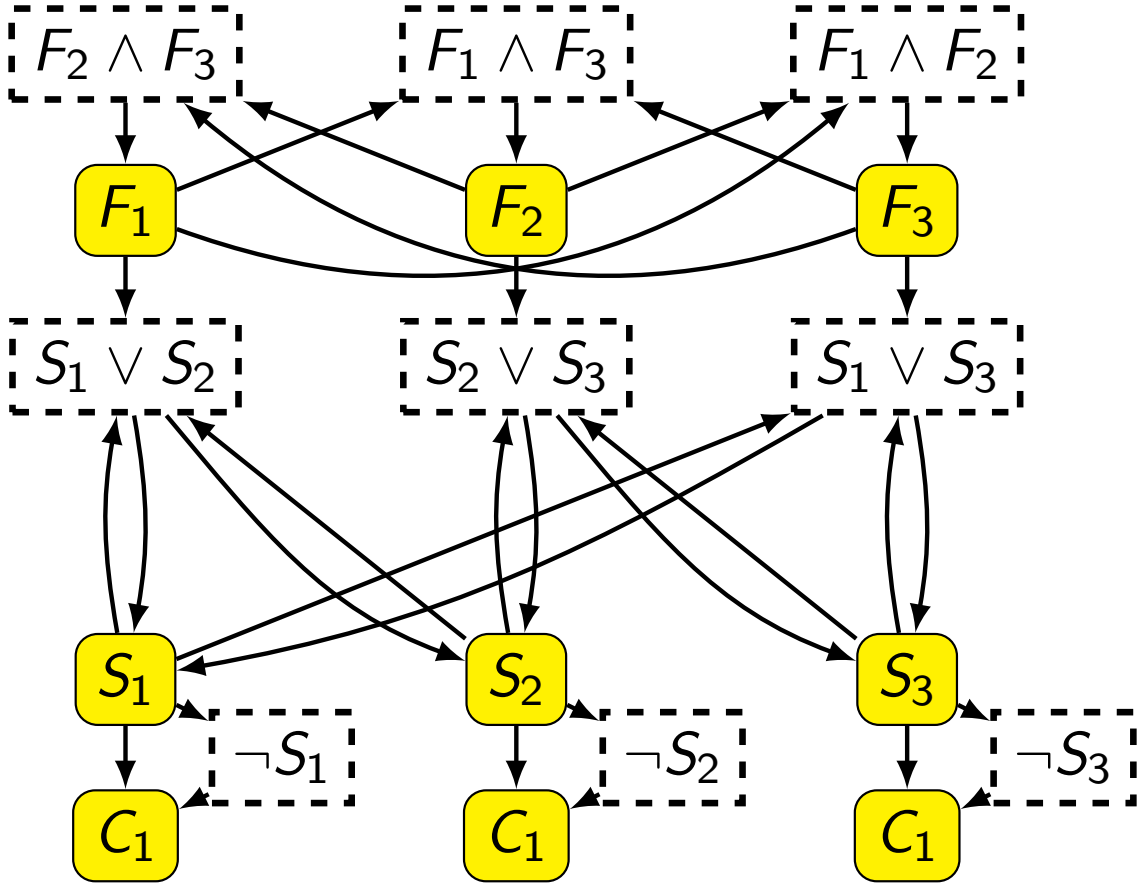
(CG)

- The *local Markov condition for DAGs*: a node $A$ is independent, given $A$'s parents $\mathrm{pa}(A)$, of all its non-descendants non-parents except $A$ itself.
- Then:

$$\mathbb{P}(X = x) = \prod_{N \in \mathcal{N}} \mathbb{P}\left(N = x_N | \mathrm{pa}(N) = x_{\mathrm{pa}(N)}\right).$$

# Logical Credal Networks

$$0.5 \leq \mathbb{P}(F_i|F_j \wedge F_k) \leq 1, \quad i \neq j, i \neq k, j \neq k;$$
$$0 \leq \mathbb{P}(S_i \vee S_j|F_i) \leq 0.2, \quad i \neq j;$$
$$0.03 \leq \mathbb{P}(C_i|S_i) \leq 0.04;$$
$$0 \leq \mathbb{P}(C_i|\neg S_i) \leq 0.01.$$

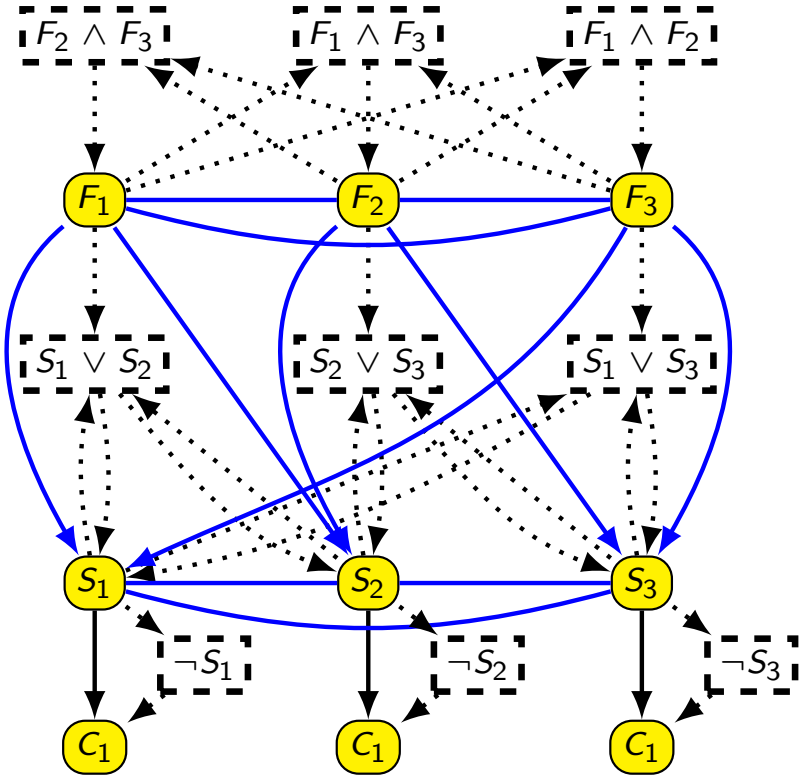


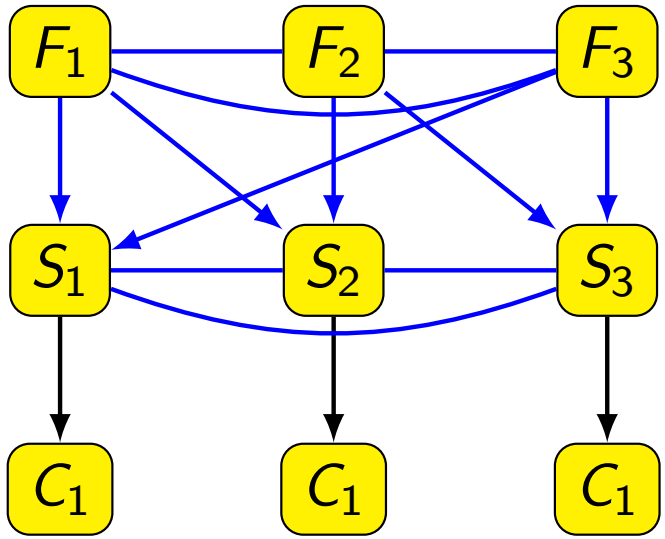
# Markov condition

## Definition (LMC(LCN))

A node $A$ is independent, given its lcn-parents, of all nodes that are not $A$ itself nor lcn-descendants of $A$ nor lcn-parents of $A$.

# The structure of a LCN

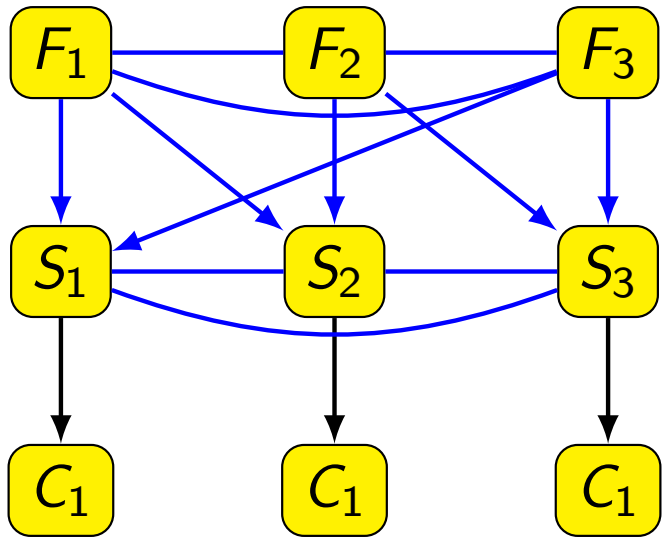



## Definition

If there is a directed path from $A$ to $B$ such that no intermediate node is a boundary node of $A$, then $B$ is a *strict descendant* of $A$.

# The Markov condition




## Definition (LMC(C-STR))

A node $A$ is independent, given its boundary, of all nodes that are not $A$ itself nor strict descendants of $A$ nor boundary nodes of $A$.

# The results

**Theorem**
*LMC(LCN) = LMC(C-STR).*

**Theorem**
*LMC(LCN) = LMC(C)*
*if the structure of a LCN is a chain graph and probabilities are positive.*

# Directed cycles: in the paper...

- Studied in connection with feedback, causality (many results, many options...).
- Possible semantics: apply the global Markov condition to the structures of LCNs and generate factorization.

# Conclusion

- LMC(LCN) can be translated to LMC(C-STR) over structures.
  - When the structure is a chain graph, we get the LMC(C) and factorization (with positivity assumption...).

- Other semantics are possible, and are worth investigating.
  - Maybe investigate bi-directed edges in "chain" graphs? Maybe investigate specification languages? Connections with causality representation?