The World Cup Football Revisited

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1. THE COMPETITION

- We asked participants to give probability intervals for the outcomes of the football matches of the World Cup of Football in Qatar.
- When the intervals $(\underline{p_1}, \overline{p_1}), (\underline{p_2}, \overline{p_2})$ assigned by two participants for a result were disjoint, with $\overline{p_1} < \underline{p_2}$, the participants played for

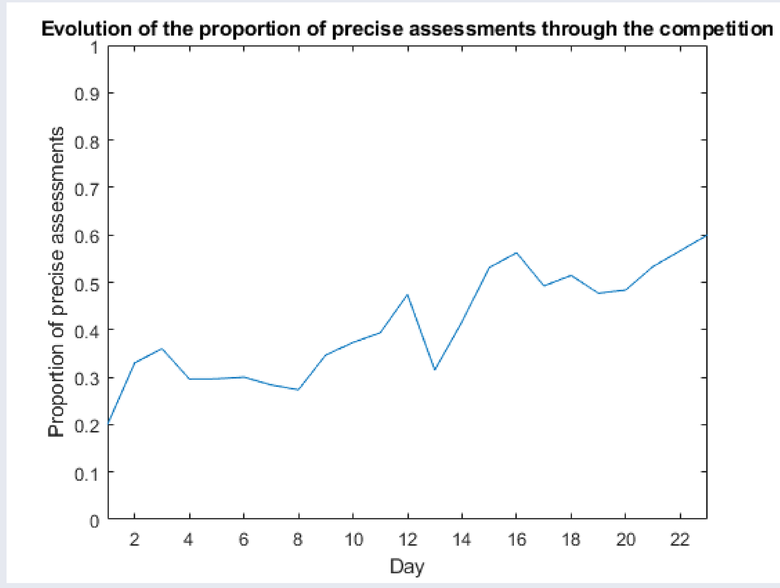
$$(\underline{p_2} - \overline{p_1}) \cdot \left|\frac{\underline{p_2} + \overline{p_1}}{2}\right| \quad \text{points,}$$

that went to the participant that went closer to the outcome that happened.
- Points doubled in the knockout stage, and tripled for the final.

Figure: https://worldcupcompetition2022eng.blogspot.com/

2. PARTICIPANTS

- 50 participants (43 men, 7 women).
- 2143 valid predictions, 51210 bets activated.
- 4 participants always played precise, 7 always imprecise.
- Only 7 participants had previous knowledge of imprecise probabilities.
- People played more precise as the competition evolved...



- ...but when being imprecise the margin of imprecision remained constant in ≈ 0.2.

3. STRATEGIES

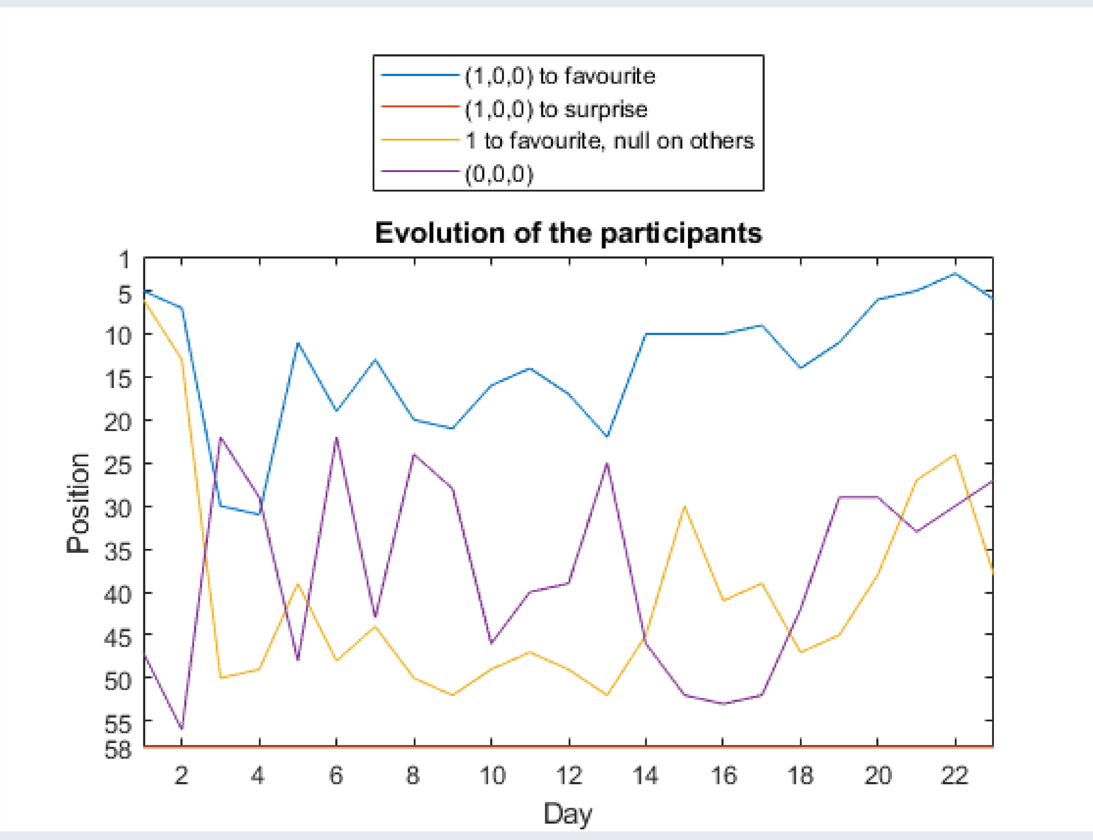


Figure: Comparison of some precise strategies

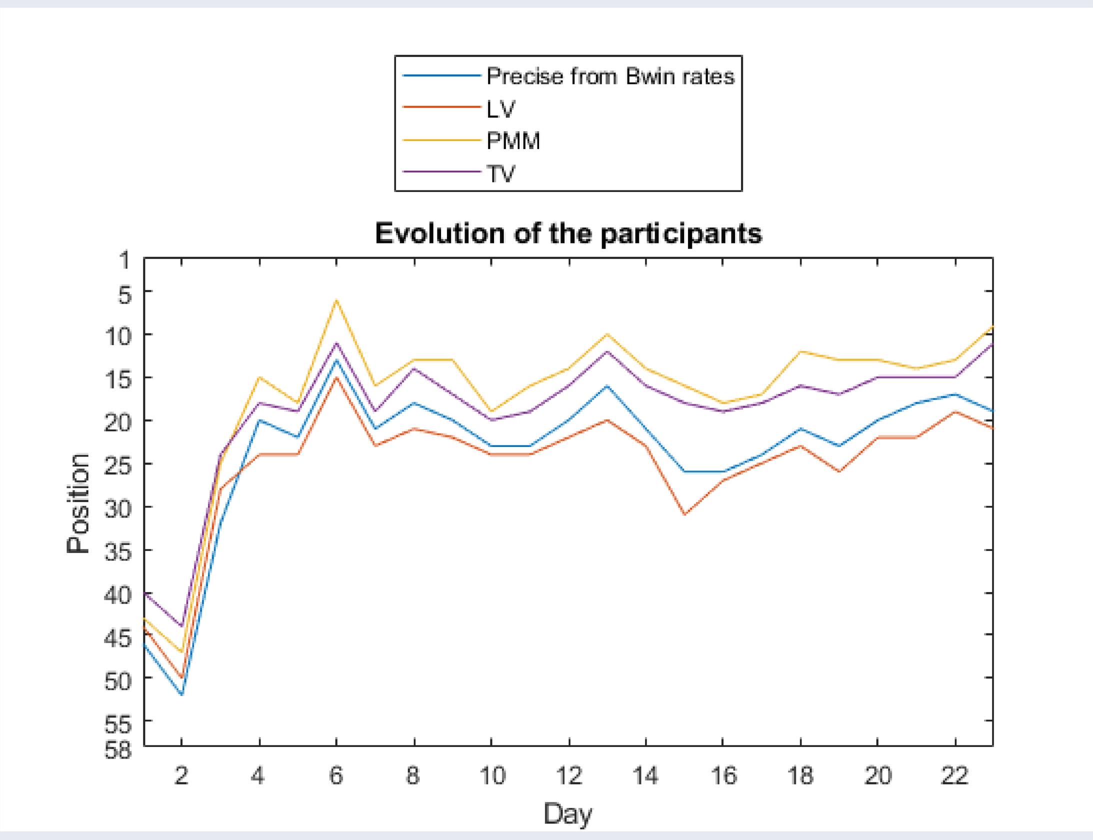


Figure: Comparison of some imprecise strategies

4. AVOIDING SURE LOSS + COHERENCE

- 78.5% of the predictions avoided sure loss.
- There were statistical significant evidences by gender and depending on whether they had previous knowledge of imprecise probabilities.
- 52.8% of the predictions were coherent.
- There were statistical significant differences by gender, field of training (Social sciences did the best!) and previous knowledge of imprecise probabilities.

5. INCOHERENCE BY SCORING RULES

We also looked at the notion of incoherence considered by Seidenfeld et al., based on de Finetti's characterisation of finitely additive probabilities.

Given an assessment $A = \{(\underline{p_1}, \overline{p_1}), (\underline{p_2}, \overline{p_2}), (\underline{p_3}, \overline{p_3})\}$, we define its **risk** as

$$\vec{R} = \left((1-\overline{p_1})^2 + \underline{p_2}^2 + \underline{p_3}^2, \underline{p_1}^2 + (1-\overline{p_2})^2 + \underline{p_3}^2, \underline{p_1}^2 + \underline{p_2}^2 + (1-\overline{p_3})^2\right)$$

and its **scoring set** as

$$\vec{S} = \{(\underline{p_1}, \underline{p_2}, \overline{p_3}), (\underline{p_1}, \overline{p_2}, \underline{p_3}), (\overline{p_1}, \underline{p_2}, \underline{p_3})\}.$$

Then we call the assessment A **incoherent of type 2** when there exists another assessment A' for the same match such that it is simultaneously:

1. *less risky* ($\vec{R}' \lneq \vec{R}$);
2. *more indeterminate*, in that the convex hull of its score $\vec{S}'$ can be embedded into that of $\vec{S}$.

- In this case, the indeterminacy can be related to the problem of embedding of triangles, for which there is a solution!.
- 90.2% of the assessments were coherent of this type.
- There were statistical significant differences by training (people without a degree did the best!) and depending on the previous knowledge of imprecise probabilities.

6. EXPLOITING THE GAME

Given an assessment $A = \{(\underline{p_1}, \overline{p_1}), (\underline{p_2}, \overline{p_2}), (\underline{p_3}, \overline{p_3})\}$, we can always give another one that guarantees not to lose points against this competitor, and maximise our possible gain.

The idea is to exploit that two of the three outcomes will not happen, and the form of the function that gives the points.

Assume w.l.o.g. $0 \leq \underline{p_1} \leq \underline{p_2} \leq \underline{p_3} \leq 1$.

We should make a precise assessment (p_1', p_2', p_3'), with:

- If $0 < \underline{p_1}$, make $p_1' = 0$, $p_2' = 0$ and $p_3' = \sqrt{\underline{p_3}^2 - \underline{p_2}^2}$ (gain guaranteed, maximises possible gain).
- If $\underline{p_1} = 0 < \underline{p_2}$, make $p_1' = 0$, $p_2' = 0$ and $p_3' = \sqrt{\underline{p_3}^2 - \underline{p_2}^2}$ (no loss guaranteed, maximises possible gain).
- If $\underline{p_1} = \underline{p_2} = 0$, make $p_i' = p_i \; \forall i \in \{1, 2, 3\}$ (no loss guaranteed).

7. CONCLUSIONS

- For this game, being precise takes you to the extreme positions.
- It usually works better to go for the favourite.
- Some ISIPTA'2023 participants: RPF (5th), NachoM (12th), Kalmanhorof (15th), ArthurVanCamp (16th), Jorgeargi19 (17th), Walley4ever (21st).

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