Indeterminacy and Imprecise Credences

Miriam Bowen

Department of Philosophy, University of St Andrews

University of St Andrews

Norms for Beleif

There are a variety of norms that purport to govern what attitude an agent ought to adopt depending on chance information, evidence available to the agent, accuracy considerations and so on.

Truth: *If an agent is certain a proposition is true she ought to believe, or be confident, or have high credence[a] in that proposition.*

Falsity: *If an agent is certain a proposition is false she ought to disbelieve, be unconfident, or have low credence[b] in that proposition.*

What should we believe about **indeterminate** propositions?

Intersectionist interpretation

Let $Cr = \{C_1, C_2, C_3, \ldots, C_i\}$ be a credal set where each of the C_i are credence functions (precise probability functions).

Broadly speaking an intersectionist interpretation takes the credal set as a whole to represent an agent's belief. That is, the credal set represents the agent's determinate attitude.

In this interpretation, we are interested in an agent's **compartive belief ordering**. That is, information of the form A is more likely than B, A and B are equally likely etc.

Kaplan [2] gives a version of intersections he calls Modest Probabilism.

(i) you are equally confident in A as you are in B if and only if every member of C assigns A the same value as it assigns B;
(ii) you are more confident in A than you are in B if and only if every member of C assigns A at least as great a value as it assigns B, and at least one member of C assigns A a greater value than it assigns B; and
(iii) otherwise you are undecided as to the relative credibility of A and B.

Examples:

Example 1:

If we consider the credal set of agent α,
$C_\alpha = \{Cr_i, Cr_j\}$ where $Cr_i(A) \geq Cr_i(B)$ and $Cr_j(A) \geq Cr_j(B)$. Then by the above interpretation $A \succsim B$, $A \succ B$ and $A \nsim B$.

Example 2:

If we consider the credal set $C_\alpha = \{Cr_k, Cr_l\}$ where $Cr_k(A) > Cr_k(B)$ and $Cr_l(A) < Cr_l(B)$. Then by the above interpretation $A \nsim B$, $A \nsucc B$ and $B \nsucc A$ so α is undecided about the relative likelihood of A and B.

Supervaluationist Interpretation

Let $Cr = \{C_1, C_2, C_3, \ldots, C_i\}$ be a credal set where each of the C_i are credence functions. Each of the credence functions in the set are permissible precisifications of the agent's belief state. It is indeterminate which of these credence functions represents the agent's beliefs.

We can say that if for a proposition A, $C_i(A) = x$, $\forall C_i \in Cr$ then the agent's credence in A is determinately x. i.e. if a proposition is assigned the same credence by every credal function in the credal set, then the agent has a determinate attitude towards this proposition.

Examples:

Agent A:

$$C_B = \{Cr_{b1}, Cr_{b2}, Cr_{b3}\}$$

where

$$Cr_{b1}(q) = 0.6, Cr_{b2}(q) = 0.7, Cr_{b3}(q) = 0.75$$

and agent B:

$$C_C = \{Cr_{c1}, Cr_{c2}\}$$

where

$$Cr_{c1}(q) = 1, Cr_{c2}(q) = 0$$

For Agent A not all credence functions in her credal set agree so this represents that she has indeterminate beliefs towards q. We can still interpret the set as giving some representation such as agent A has credence higher than 0.6 in q. For agent B there is no consensus between her credence functions- she has maximally indeterminate beliefs.

Indeterminacy

Williams [3] presents a variety of cases that he notes many have categorized as cases of indeterminacy. A selection of these are as follows:

(1) Will the flipped coin currently spinning in the air land heads?
(2) Is the Liar sentence true?
(4) Is Patchy red?
(7) Is the King of France bald?
(8) Is *The Turn of the Screw* a ghost story?
(10) Is this superposed particle spin-up?

[a] *Where high credence is understood as having as having a credence greater than* 0.5.
[b] *Where low credence is understood as having a credence less than* 0.5.

Indeterminacy

Different categories of indeterminacy? Metaphysical indeterminacy, epistemic indeterminacy and semantic (or referential) indeterminacy?

- **Metaphysical indeterminacy**: E.g. quantum phenomena or phenomena involving the open future
- **Referential indeterminacy**: E.g. When Newtonian's were talking about 'mass'. Einstein showed there are two things Newtonian 'mass' can refer to: proper mass and relativistic mass.

But...

- Is there a uniform kind of indeterminacy within each of these categories?
- There is potential disagreement about what type of indeterminacy is present in certain domains, but agreement that there is *indeterminacy* [e.g **vagueness**].

Normative Silence

. . . so far as general alethic norms go there are simply no constraints on what the Godlike attitude to p should be, when p is indeterminate. [3, p.223]

Williams argues for a strong version of this claim: Not only is there no unique norm that governs what attitude an agent ought to adopt in cases of indeterminacy, but also that *any* cognitive role of indeterminacy can be adopted.

Bridge Principles

We need some kind of bridge principle between logical validity and its conceptual role in the regulation of belief.

For example:

"(VP) Our degrees of belief should (non-subjectively) be such that

(2) If $A_1, \ldots A_n \Rightarrow B$ then $Cr(B) \geq \sum_i Cr(A_i) - n + 1$

To make this less opaque, let's introduce the abbreviation Dis(A) for $1 - Cr(A)$; we can read Dis as "degree of disbelief". Then an equivalent and more immediately compelling way of writing (2) is

(2equiv) If $A_1, \ldots A_n \Rightarrow B$ then Dis(B) $\leq \sum_i$ Dis(A_i)". [1, p.45]

Modest Pluralism

- Modest pluralism retains the idea that there is no unique attitude that an agent ought to adopt towards a proposition in virtue of being certain it is indeterminate.
- There are restrictions on what attitudes it is permissible for an agent to adopt towards any proposition.

So what logic underlies the permissible attitudes? Broadly classic.

The permissible types of attitudes rational agents can adopt are:

- Precise probabilities/ precise credences. A function $C(.)$ that satisfies:
  - Normalisation For any logical truth $\top$ $C(\top) = 1$.
  - Non-negativity For any proposition ϕ, $0 \leq C(\phi)$.
  - Finite additivity If ϕ and ψ are incompatible propositions, then $C(\phi \vee \psi) = C(\phi) + C(\psi)$.
- Credal sets: A set $Cr = \{C_1, C_2, C_3, \ldots, C_i\}$ where each of the C_i are credence functions.
- Sets of credal sets: A set $Cr = \{Cr_1, Cr_2, Cr_3, \ldots, Cr_i\}$ where each of the Cr_i are credal sets.

Specifically, we can think of the interpretations (intersectionist and supervaluationist) as applying to the different types of permissible attitudes.

- Credal sets: intersectionist interpretation.
- Sets of credal sets: supervaluationist interpretation.

This allows us to represent that an agent might have indeterminate attitudes about what comparative belief ordering they take to be representative of attitude towards a particular proposition.[a]

References

[1] Hartry Field.
What is logical validity?
In Colin R. Caret and Ole T. Hjortland, editors, *Foundations of Logical Consequence*. Oxford University Press, 2015.

[2] Mark Kaplan.
In Defense of Modest Probabilism.
Synthese, 176(1):41–55, 2010.

[3] J. Robert G. Williams.
Indeterminacy and normative silence.
Analysis, 72(2):217–225, 2012.

[a] A slight adaptation of the supervaluationist position might be appropriate here such that this view does not reduce to representing an agent's attitude as merely the weakest comparative ordering in the set. Rather, we might limit what can be read off from the set in the supervaluationist interpretation as applied to sets of credal sets.