Markov Conditions and Factorization in Logical Credal Networks

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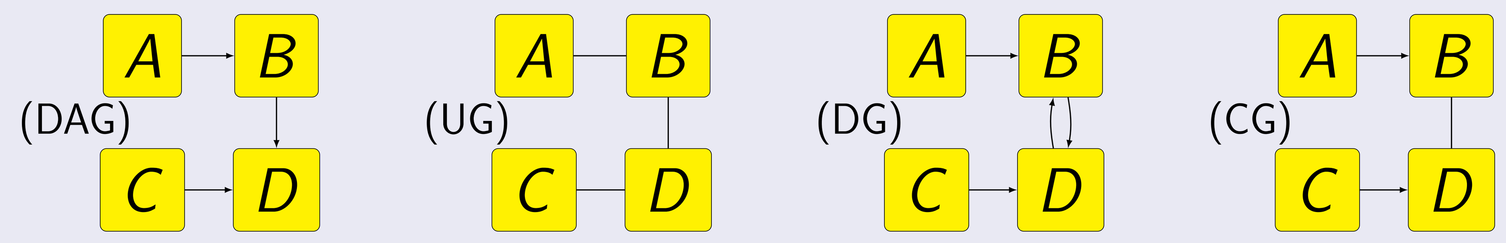
Goal:

- We examine the recent language of Logical Credal Networks, in particular the consequences of various Markov conditions.
- We introduce the notion of structure for a Logical Credal Network. Result: structure without directed cycles = factorization.
- For networks with directed cycles, there are differences in Markov conditions, factorization results, specification requirements.

Graphs...



Markov conditions for chain graphs

Definition (LMC(C))

A node A is independent, given its parents, of all nodes that are not A itself nor descendants nor boundary nodes of A.

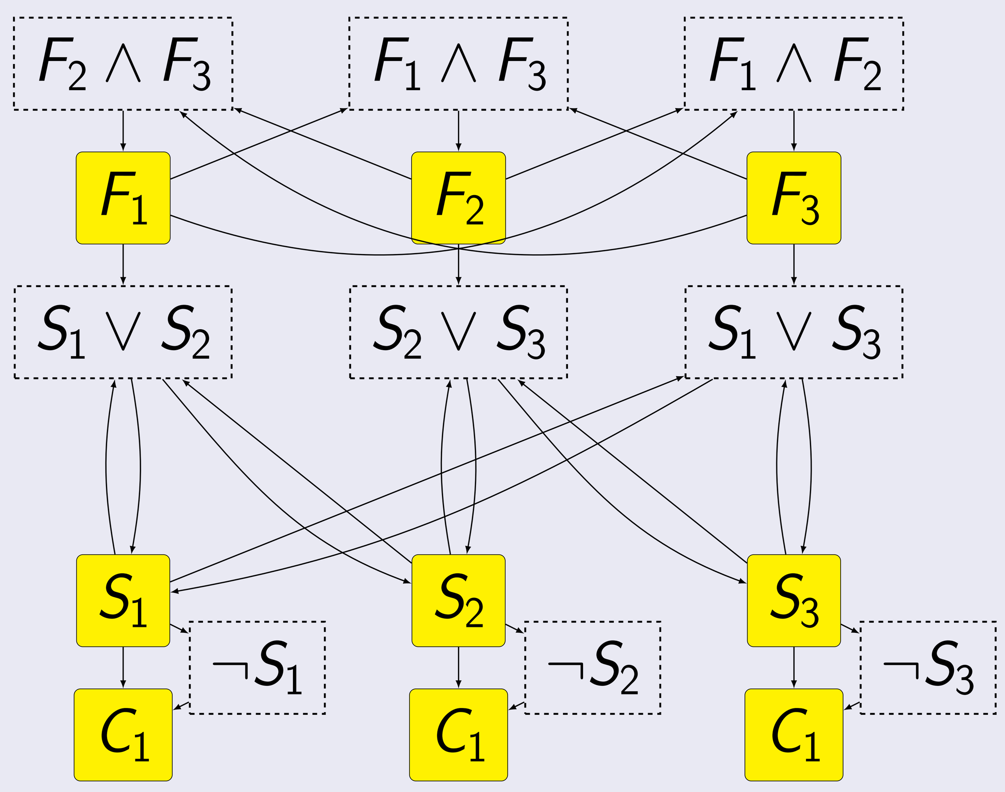
For Bayesian networks, consequence:
$\mathbb{P}(X = x) = \prod_{N \in \mathcal{N}} \mathbb{P}(N = x_N | \mathrm{pa}(N) = x_{\mathrm{pa}(N)})$.

Definition (GMC(C))

Given any triple $(\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3)$ of disjoint subsets of $\mathcal{N}$, if $\mathcal{N}_2$ separates $\mathcal{N}_1$ and $\mathcal{N}_2$ in the graph $\mathcal{G}^{ma}(\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3)$, then nodes $\mathcal{N}_1$ and $\mathcal{N}_3$ are independent given nodes $\mathcal{N}_2$.

Logical Credal Networks

$$0.5 \leq \mathbb{P}(F_i | F_j \wedge F_k) \leq 1, \; i \neq j, i \neq k, j \neq k;$$
$$0 \leq \mathbb{P}(S_i \vee S_j | F_i) \leq 0.2, \;\; i \neq j;$$
$$0.03 \leq \mathbb{P}(C_i | S_i) \leq 0.04;$$
$$0 \leq \mathbb{P}(C_i | \neg S_i) \leq 0.01.$$



Definition

The *lcn-parents* of a proposition A are the propositions such that there exists a directed path in the dependency graph from each of them to A in which all intermediate nodes are formulas.

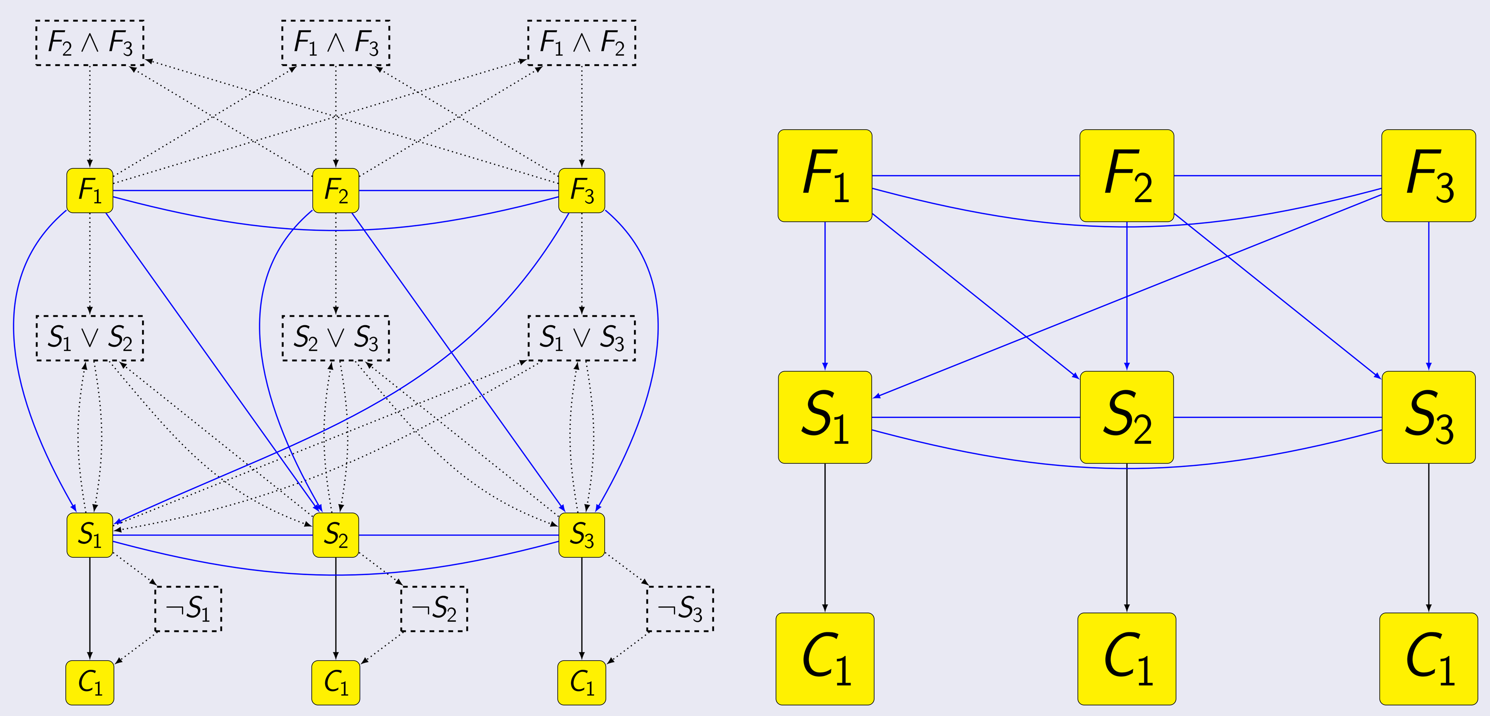
Definition

The *lcn-descendants* of a proposition A are the propositions such that there exists a directed path in the dependency graph from A to each of them in which no intermediate node is a parent of A.

Definition (LMC(LCN))

A node A is independent, given its lcn-parents, of all nodes that are not A itself nor lcn-descendants of A nor lcn-parents of A.

The structure of a LCN



Results

Lemma

The set of lcn-parents of a proposition A in a LCN is identical to the boundary of A with respect to the structure of the LCN.

Definition

If there is a directed path from A to B such that no intermediate node is a boundary node of A, then B is a *strict descendant* of A.

Definition (LMC(C-STR))

A node A is independent, given its boundary, of all nodes that are not A itself nor strict descendants of A nor boundary nodes of A.

Theorem

Given a LCN, the Markov condition LMC(LCN) is identical, with respect to the independence relations it imposes, to the local Markov condition LMC(C-STR) applied to the structure of the LCN.

Theorem

If the structure of a LCN is a chain graph, and probabilities are positive, then the Markov condition LMC(LCN) is identical, with respect to the independence relations it imposes, to the LMC(C) applied to the structure.

Directed cycles: more in the paper...

Difference between LMC(C) and GMC(C), weakness of LMC(C), possible factorizations; proposal: apply GMC(C) to structures of LCNs.

Conclusion

- LMC(LCN) can be translated to LMC(C) over structures.
- When the structure is a chain graph, we get the usual factorization (with positivity assumption...).
- Other semantics are possible, and may be worth investigating (connections with causality).
- How about cycles? Maybe investigate bi-directed edges in "chain" graphs? Maybe investigate specification languages?

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