Removing Redundancies for Faster Inference

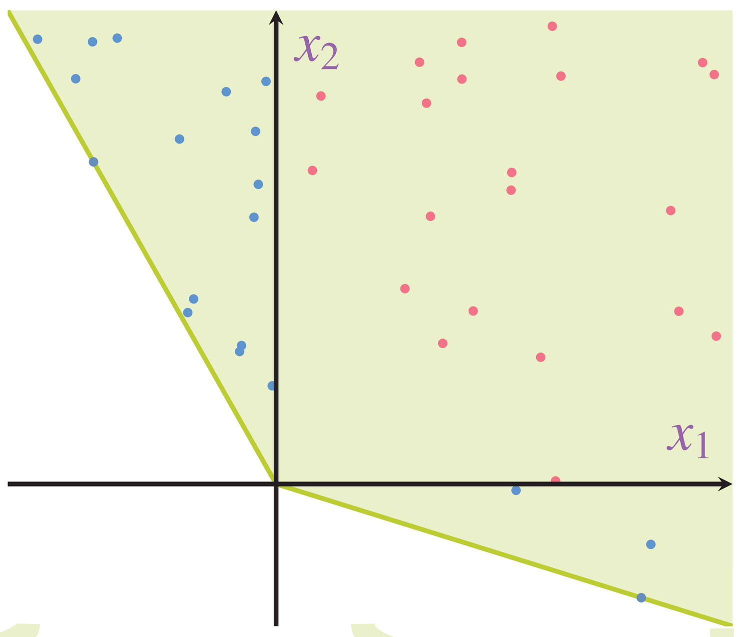
GHENT UNIVERSITY

Arne Decadt
FLip

$D = \{d_1, \dots, d_m\} \subset \mathbb{R}^{\mathscr{X}} \sim \mathbb{R}^n$ **desirable** gambles

positive = uninformative = removable

How to remove more redundancies?



$g \in$ **natural extension** $\mathscr{E}(D)$

$\Updownarrow$

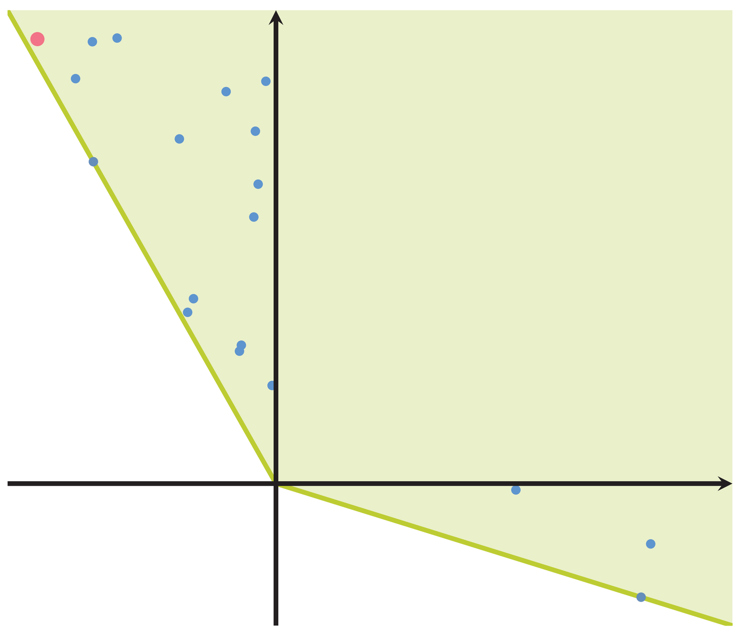
$g \neq 0$ and $\exists \lambda_1 \geq 0, \dots, \lambda_m \geq 0$

$$g \geq \sum_{k=0}^{m} \lambda_k d_k = \underbrace{(d_1 | \cdots | d_m)}_{\mathbf{D}} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_m \end{pmatrix}$$

$\rightarrow$ linear programming

What do the E-vectors represent when interpreted as maps from $\mathscr{X}$ to $\mathbb{R}$?

Naive



If $d \in \mathscr{E}(D \setminus \{d\})$, remove d from D

Check every $d \in D$

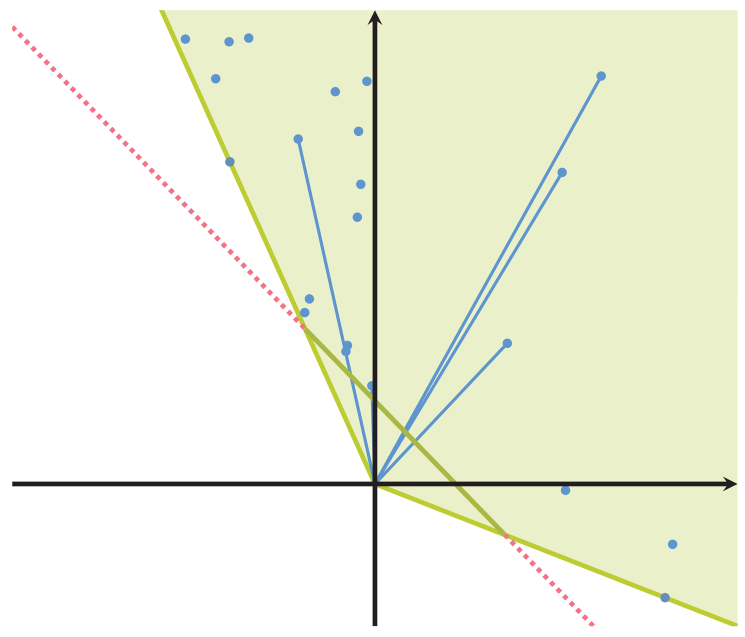
Difficulty?

m linear programs of size $n \times m$

Where used?

Standard in many redundancy removal packages such as CDDLIB and POLYHEDRA.JL [2]

Hyperplane



① Find a hyperplane that intersects $\mathscr{E}(D)$
linear feasibility problem:

find	$v \in \mathbb{R}, v \geq 1$
s.t.	$\mathbf{D}^{\mathrm{T}} v \geq 1$

Hyperplane H: $g \in H \Leftrightarrow v \cdot g = 1$

② Scale all d_k and all $\mathbb{I}_x$ onto H

$$d'_k = \frac{d_k}{v \cdot d_k}; \mathbb{I}'_x = \frac{\mathbb{I}_x}{v \cdot \mathbb{I}_x}$$

③ Remove last coordinate retrievable from $v \cdot g = 1$

$$d''_k; \mathbb{I}''_x$$

④ Find convex hull on H

n	Method
2	min & max
3,4	$O(m \log m)$ algorithm
$5, \dots, \sim 8$	QHull
higher	Naive Method

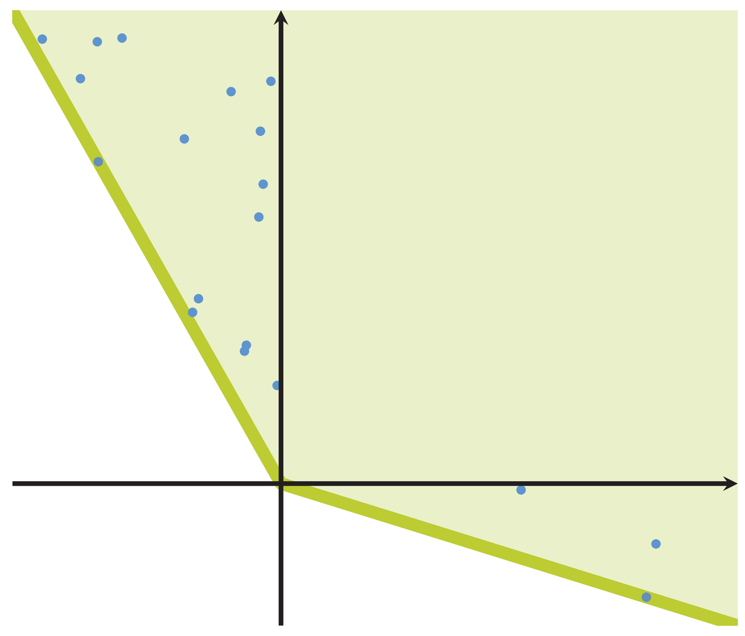
Difficulty?

One linear program of size $m \times n$

+

Difficulty of the convex hull
$\rightarrow$ exponential in n

Dual Description



① Use expectations = hyperplanes
For every linearly independent
$\{g_1, \dots, g_{n-1}\} \subseteq D \cup \{\mathbb{I}_x \colon x \in \mathscr{X}\}$

$$\mathrm{E} = \begin{pmatrix} \vdots \\ g_{n-1}^{\mathrm{T}} \\ \mathbf{1}^{\mathrm{T}} \end{pmatrix}^{-1} \begin{pmatrix} \vdots \\ 0 \\ 1 \end{pmatrix}$$

② Remove redundant E's
linear feasibility problem:

find	$\mu \in \mathbb{R}^{\ell-1}, \mu \geq 0, \sum \mu = 1$
s.t.	$(\cdots \lvert \mathrm{E}_{k-1} \lvert \mathrm{E}_{k+1} \rvert \cdots \lvert \mathrm{E}_\ell)\, \mu = \mathrm{E}_k$

If feasible $\rightarrow$ remove E_k

③ Inference using lower prevision
$g \in \mathscr{E}(D) \Leftrightarrow g \neq 0 \wedge \min_{\mathrm{E}} \mathrm{E} \cdot g \geq 0$

Difficulty?

After simplification,
tight upper bound on ℓ [1, p. 394-395]

$$\ell \leq \binom{m - \lceil n-1/2 \rceil}{\lfloor n-1/2 \rfloor} + \binom{m - \lceil n-1/2 \rceil - 1}{\lfloor n-1/2 \rfloor - 1}$$

$\rightarrow$ exponential in n and often achieved in random experiments
$\Rightarrow$ exponential memory required
$\Rightarrow$ calculate exponential # inner products

References

[1] Martin Henk, Jürgen Richter-Gebert, and Günter M Ziegler. Basic properties of convex polytopes. In *Handbook of discrete and computational geometry*, pages 383–413. Chapman and Hall/CRC, 2017.

[2] Benoît Legat. personal communication.

[3] Nawapon Nakharutai, Matthias CM Troffaes, and Camila CS Caiado. Improved linear programming methods for checking avoiding sure loss. *International Journal of Approximate Reasoning*, 101:293–310, 2018.