Eliciting hybrid probability-possibility functions and their decision evaluation models

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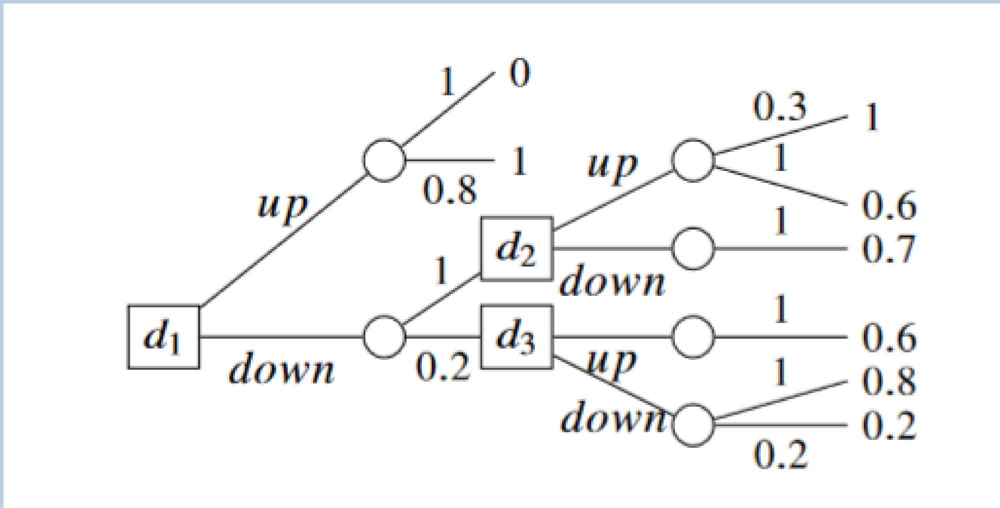
Sequential decision problems



Three assumptions are instrumental to enable an optimal strategy to be computed using dynamic programming:

- *Dynamic Consistency*: when following an optimal strategy and reaching a decision node, the best decision at this node is the one that had been considered so when computing this strategy, i.e. prior to applying it.
- *Consequentialism*: the best decision at each step of the problem only depends on potential consequences at this point.
- *Tree Reduction*: a compound lottery is equivalent to a simple one, assigning probabilities to final states.

Hybrid prob-poss measures

A hybrid π-p measures that combine probabilistic and possibilistic behaviors in the uncertainty context.

$$\rho^{\alpha}(s) = \alpha\pi(s) + (1-\alpha)p(s), \quad \alpha \in [0,1]$$

where p and π satisfy the constraint $p(s) = 0$ if $\pi(s) < 1$ for all s. ρ^{α} is a possibility distribution if $\alpha = 1$, a probability distribution if $\alpha = 0$. Note that $1 \leq \sum_{s\in S} \rho^{\alpha}(s) \leq n$.

Hybrid distributions generate a class of decomposable capacities, which are monotonic set functions such that if $A \cap B = \emptyset$:

$$\rho^{\alpha}(A \cup B) = S^{\alpha}(\rho^{\alpha}(A), \rho^{\alpha}(B))$$

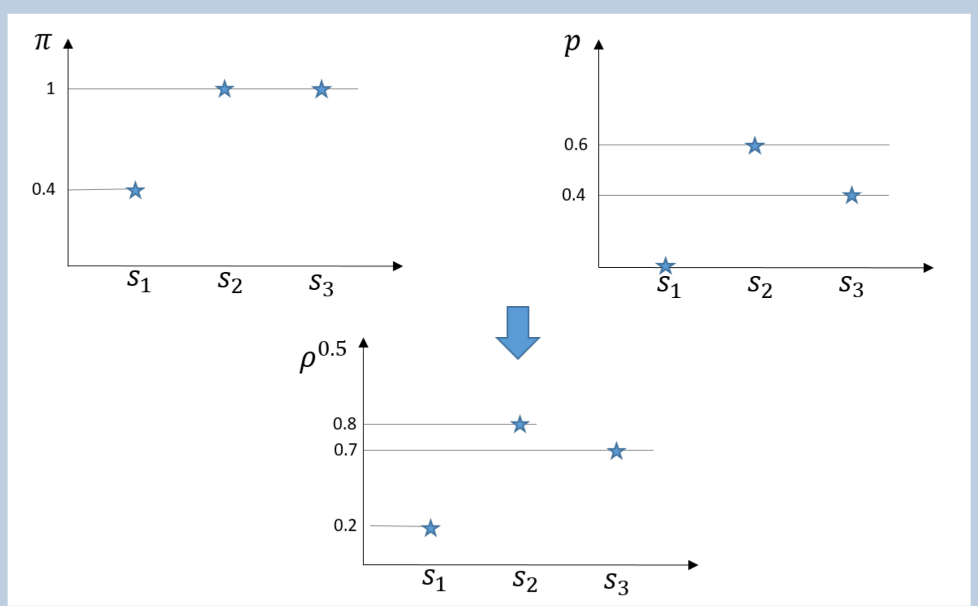
$$S^{\alpha}(x,y) = \begin{cases} \min(1, x+y-\alpha) \text{ if } x > \alpha, y > \alpha \\ \max(x,y) \text{ otherwise,} \end{cases}$$

In order to reduce probability-possibility lotteries, an operation $*$ is needed to generalize probabilistic independence. If A and B are disjoint sets independent of another set C:

$$\begin{aligned}\rho^{\alpha}((A \cup B) \cap C) &= S^{\alpha}(\rho^{\alpha}(A), \rho^{\alpha}(B)) * \rho^{\alpha}(C) \\ &= S^{\alpha}(\rho^{\alpha}(A) * \rho^{\alpha}(C), \rho^{\alpha}(B) * \rho^{\alpha}(C)).\end{aligned}$$

This distributivity property is valid only when the operation $*$ is a triangular norm of the form

$$x *_{\alpha} y = \begin{cases} \alpha + \frac{(x-\alpha)(y-\alpha)}{1-\alpha} \text{ if } x > \alpha, y > \alpha \\ \min(x,y) \text{ otherwise.} \end{cases}$$

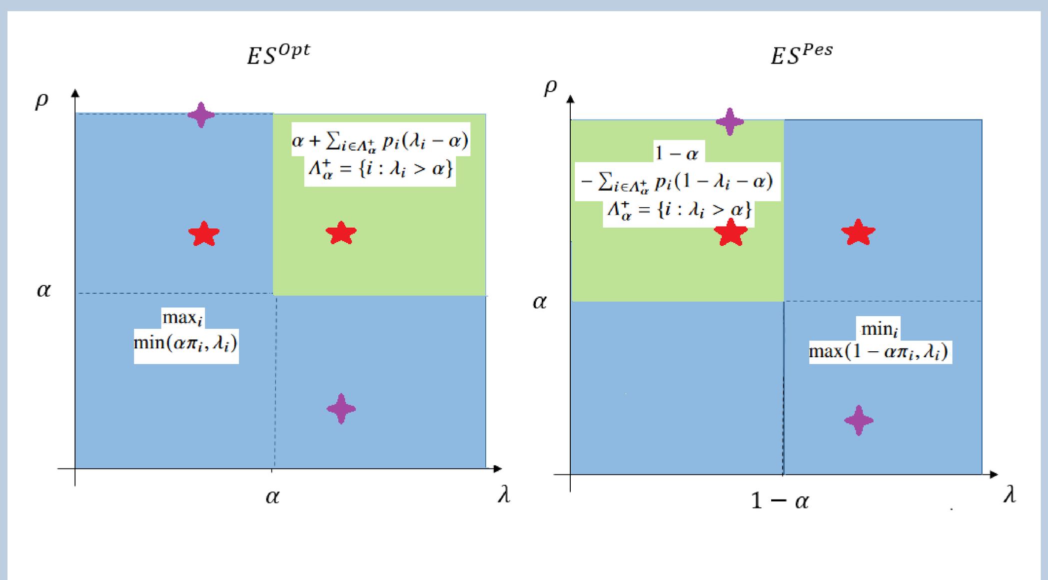


Example 1 $\rho^{0.5}(\{s_1,s_2\}) = \max(0.2, 0.8) = 0.8$, $\rho^{0.5}(\{s_1,s_3\}) = \max(0.2, 0.7) = 0.7$, $\rho^{0.5}(\{s_2,s_3\}) = 0.8 + 0.7 - 0.5 = 1$.

$\Rightarrow$ *This distribution defines a convex set of probability distributions. We can express this probability set by inequalities:*

$P(\{s_1,s_2,s_3\}) = 1$, $0 \leq P(\{s_1\}) \leq 0.2$, $0.3 \leq P(\{s_2\}) \leq 0.8$, $0.2 \leq P(\{s_2\}) \leq 0.7$, $0.3 \leq P(\{s_1,s_2\}) \leq 0.8$, $0.2 \leq P(\{s_1,s_3\}) \leq 0.7$ *and* $0.3 \leq P(\{s_2,s_3\}) \leq 1$.

Utility functionals



Example 2 *Situation a has utility* $\lambda_a = 0.3$ *and b* $\lambda_b = 0.7$. $\alpha = 0.5$

- *Decision 1:* $\pi_a = \pi_b = 1$, *and* $p_a = p_b = 0.5$
- *Decision 2:* $\pi_a = 1, \pi_b = 0.2$, *and* $p_a = 1, p_b = 0$.
- $ES^{Opt}(D1) = 0.5 + 0.5 * (0.7 - 0.5) = 0.6 \succ_{ES^{Opt}} ES^{Opt}(D2) = max(min(0.5, 0.3), min(0.1, 0.7)) = 0.3$
- $ES^{Pes}(D1) = 0.5 - 0.5(1 - 0.3 - 0.5) = 0.4 \succ_{ES^{Pes}} ES^{Pes}(D2) = 0.5 - (1 - 0.3 - 0.5) = 0.3$

Elicitation of a prob-poss model from given weights

Example 3 *Consider two distributions on* $S = \{a, b\}$

- *1:* $\rho_a = \rho_b = 0.6$
- *2:* $\rho'_a = \rho'_b = 0.5$

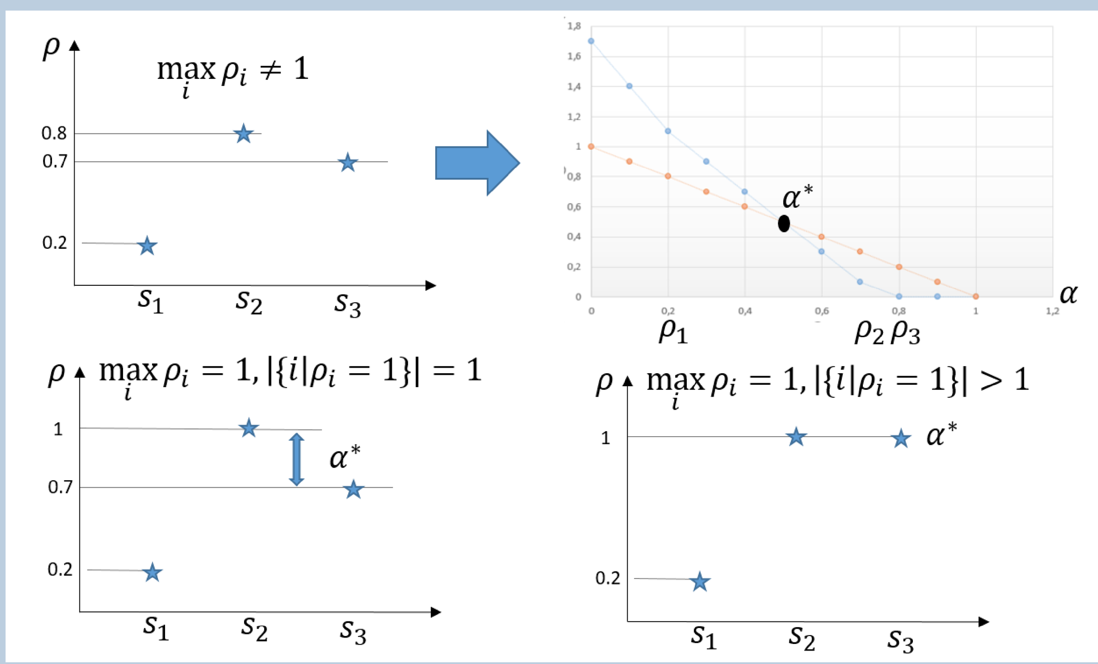
We can see that renormalizing these distributions in agreement with possibility or probability, the resulting two distributions 1 and 2 are the same:

- $p^1_a = p^2_a = 0.5 = p'^1_b = p'^2_b$
- $\pi'^1_a = \pi'^2_a = 1 = \pi'^1_b = \pi'^2_b$

Hence no distinction between 1 and 2 can be made using this kind of transformation. Using the hybrid interpretation

- *Case 1: with* $\alpha = 0.2$, $\pi_a = \pi_b = 1$, *and* $p_a = p_b = 0.5$ *we can check that* $\rho_a = \rho_b = 0.2 + 0.8 \cdot 0.5$, *(a mixture between uniform probabilities and possibilities).*
- *Case 2:* $\alpha = 0$, $\pi_a = \pi_b = 1$, *and* $p_a = p_b = 0.5 = \rho_a = \rho_b$, *(a pure probability distribution).*

The question is: given a distribution of weights $(\rho_1, \dots \rho_n) \in [0,1]^n$ on S such that $\sum_{i=1}^{n} \rho_i \geq 1$, does there exist a threshold $\alpha \in [0,1]$, a possibility distribution π and a probability distribution p on S, such that $\rho = \alpha\pi + (1-\alpha)p$? If yes, is the 3-tuple (α, π, p) uniquely defined?

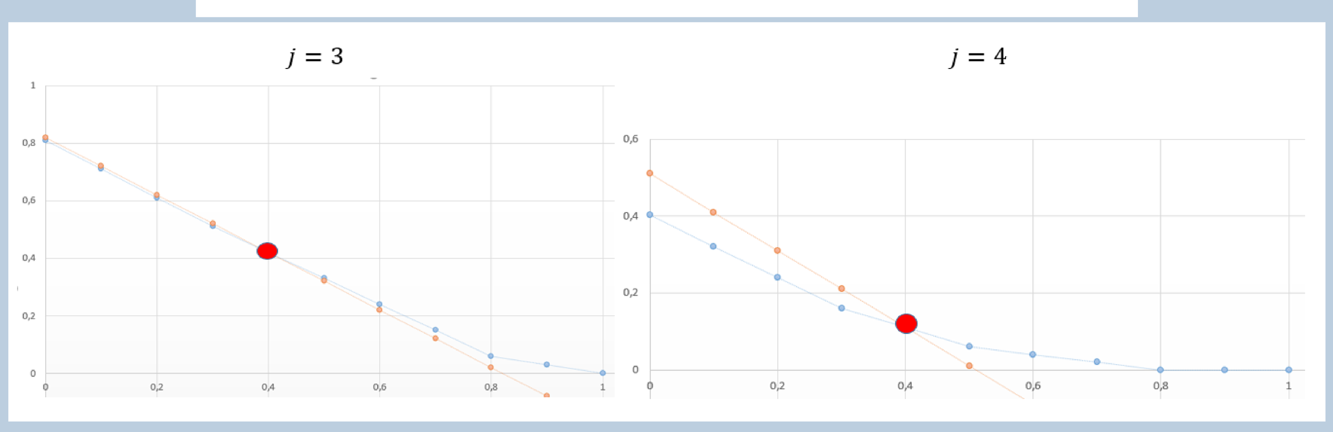


Elicitation from global ratings of loteries

The dataset is a set of tuples (π^j, p^j, β^j) $j \in J = \{1, ..., m\}$ where π^j is a possibility distribution, p^j is a probability distribution j is a strategy, and β^j is the global evaluation given by an expert.

	i : λ_i	1 0.01	2 0.3	3 0.5	4 0.8	5 1	β
$j=1$	π^1	0.2	0.6	1	1	0	0.8
	p^1	0	0	0	1	0	
$j=2$	π^2	1	1	0.5	0.5	0	0.3
	p^2	0.4	0.6	0	0	0	
$j=3$	π^3	0	1	0.5	1	1	0.82
	p^3	0	0.1	0	0.6	0.3	
$j=4$	π^4	1	1	1	1	1	0.51
	p^4	0.2	0.3	0.3	0.2	0	

	$\underline{\alpha}$	$\overline{\alpha}$
$j=1$	0	1
$j=2$	0.3	0.6
$j=3$	0	0.82
$j=4$	0	0.51
α	0.3	0.51



The decision-maker is consistent across all four examples: $\alpha = 0.4$ is a valid choice for the 4 items.