Robust possibilistic production planning under temporal demand uncertainty with knowledge on dependencies

R.Guillaume
romain.guillaume@irit.fr

Production planning problem

Our problem can be modeled by the following linear program:

$$\min \sum_{t\in[T]} (c^I I_t + c^B B_t + c^P x_t - b^P s_t)$$
$$\text{s.t. } B_t - I_t = D_t - X_t \quad t \in [T],$$
$$\sum_{i\in[t]} s_i = D_t - B_t \quad t \in [T],$$
$$B_t, I_t, s_t \geq 0, \boldsymbol{x} \in \mathbb{X} \subseteq \mathbb{R}_+^T \quad t \in [T]$$

where $D_t = \sum_{i\in[t]} d_i$ and $X_t = \sum_{i\in[t]} x_i$, D_t and X_t stand for the cumulative demand up to period t and the cumulative production up to period t, respectively. I_t and B_t are respectively the inventory and the backordering à period t

Temporal demand uncertainty

let $\hat{d}_t$ be the forecasting demand at period t and $\hat{D}_t = \sum_{i\in[t]} \hat{d}_i$ the cumulative forecasting demand. The manager can give for each period $t \in [T]$ the maximum percent of demand $\delta_t^- \in [0,1]$ that can be advanced to the previous period $t-1$ and maximum percent of demand $\delta_t^+ \in [0,1]$ that can be delayed to the next period $t+1$. The possibility distribution $\pi_{\tilde{\mathbf{D}}}$ is defined in terms of $\lambda - cut$: $\mathbf{D}^{[\lambda]} = \{\mathbf{D} | \pi_{\tilde{\mathbf{D}}} \geq \lambda\}, \forall \lambda \in [0,1]$ by the following equation .

$$\begin{aligned}
\mathbf{D}^{[\lambda]}: \quad & \{\boldsymbol{D}^{[\boldsymbol{\lambda}]} \in \mathbb{R}^T \\
s.t. \quad & D_t^{[\lambda]} \geq \hat{D}_t - \beta_t^+ \delta_t^+ \hat{d}_t, \forall t \in [T] && (a)\\
& D_t^{[\lambda]} \leq \hat{D}_t + \beta_{t+1}^- \delta_{t+1}^- \hat{d}_{t+1}, \forall t \in [T] && (b)\\
& \beta_t^- \leq 1 - \lambda, \forall t \in [T] && (c)\\
& \beta_t^+ \leq 1 - \lambda, \forall t \in [T] && (d)\\
& \beta_t^+ \delta_t^+ + \beta_t^- \delta_t^- \leq 1, \forall t \in [T] && (e)\\
& \beta_t^-, \beta_t^+ \in [0,1], \forall t \in [T]\}
\end{aligned}$$

$\Pi-$RO

Robust possibilistic optimization maximize the degree of certainty that a solution cost is lower than a given threshold g.

$$\max_{\boldsymbol{x}\in\mathbb{X}} \quad N(\mathcal{C} \leq g)$$

The necessity measure is computed from the marginal possibility distribution and a function $H_\alpha : [0,1]^T \to [0,1]$ (eq.1) where $\alpha \in [0,1]$ is the degree of belief that they is negative dependencies, i.e.

- if $\alpha = 0$ we do not have negative dependencies between uncertainty,
- if $\alpha = 1$ we know that there is negative dependencies between uncertainty of each periods.

$$H_\alpha(\boldsymbol{u}) = \alpha.W(\boldsymbol{u}) + (1-\alpha)M(\boldsymbol{u}), \alpha \in [0,1] \quad (1)$$

with $W(\boldsymbol{u}) = \max\{\sum_{i=1}^T u_i - T + 1, 0\}$ and $M(\boldsymbol{u}) = \min_{i=1,\ldots,T}(u_i)$.

The necessity degree to have a vector $\boldsymbol{x} \in \mathbb{R}^T$ lower or equal to a given vector $\boldsymbol{a}$ is:

$$N(\boldsymbol{x} \leq \boldsymbol{a}) = 1 - H_\alpha(\Pi(a_1 < x_1), \ldots, \Pi(a_T < x_T))$$

Problem formulation

Proposition 1 *The marginal possibility distribution π_{D_t} is triangular possibility distribution represented by the lower possible value: $\hat{D}_t - \delta_t^+ \hat{d}_t$, the most possible value: $\hat{D}_t$ and the higher possible value: $\hat{D}_t + \delta_{t+1}^- \hat{d}_{t+1}$.*

the marginal possibility distribution on cost from the marginal possibility distribution on D_t, where $D_t^{[\lambda]}$ is the λ-cut of possiblity distribution π_{D_t}.

$$\mathcal{C}_t^{[\lambda]} = [\min_{D_t\in D_t^{[\lambda]}} \mathcal{C}_t, \max_{D_t\in D_t^{[\lambda]}} \mathcal{C}_t] = [\underline{\mathcal{C}}_t^{[\lambda]}, \overline{\mathcal{C}}_t^{[\lambda]}] \quad (2)$$

The problem of Robust possibilistic production planning under temporal uncertainty taking into account dependencies belief is:

$$\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{X}} \quad & \epsilon \\
& \max_{\{\mathcal{C}=\boldsymbol{c}|H_\alpha(\boldsymbol{c})\geq\epsilon\}} \sum_{t\in[T]} \mathcal{C}_t \leq g \\
& \boldsymbol{D} \in \boldsymbol{D}^{[0]}
\end{aligned} \quad (3)$$

Solving the adversarial problem

First we focus on the problem :

$$\begin{aligned}
\max_{\{\mathcal{C}=\boldsymbol{c}|H_\alpha(\boldsymbol{c})\geq\epsilon\}} \quad & \sum_{t\in[T]} c_t \\
& \boldsymbol{D} \in \boldsymbol{D}^{[0]}
\end{aligned} \quad (4)$$

Theorem 1 *The problem 4 can be computed in $O(T^2)$ times*

The main idea of the proof is that the problem can be formulated as longest path problem for graphs with one of the following structures :

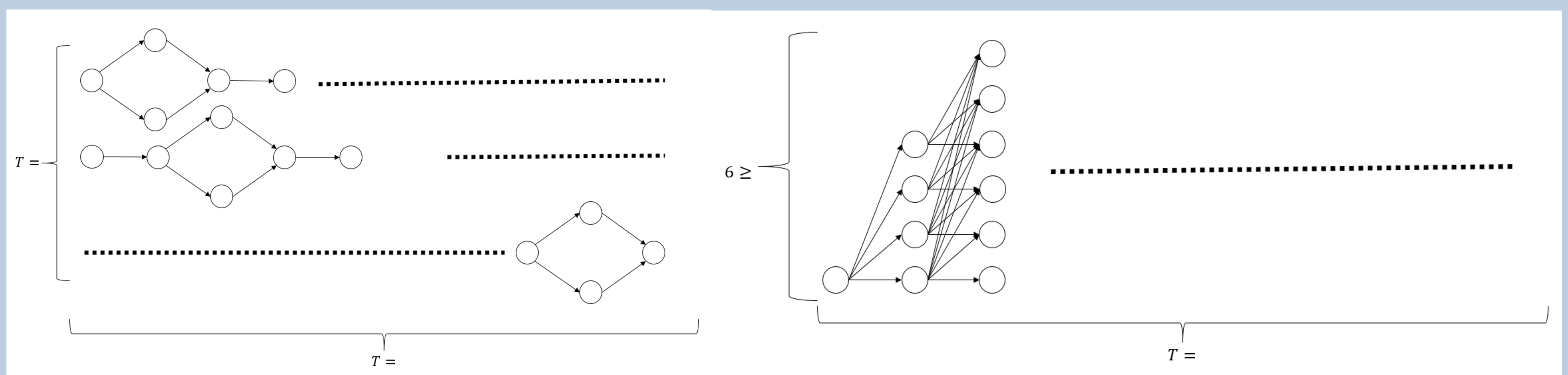
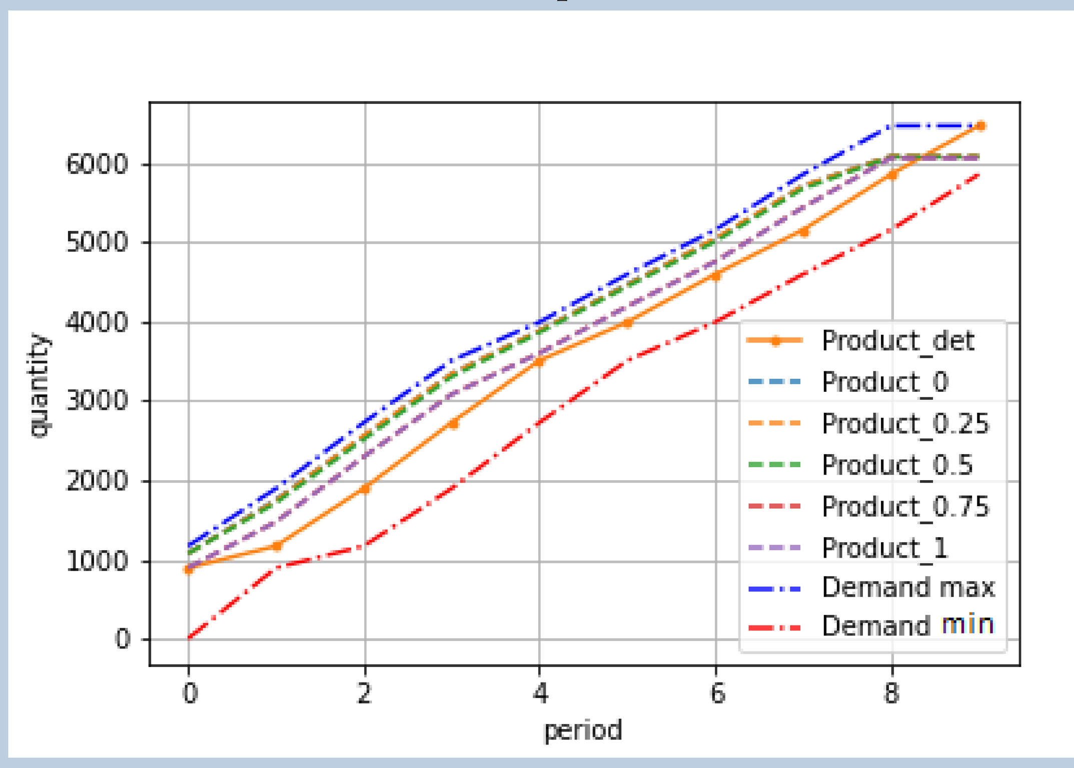


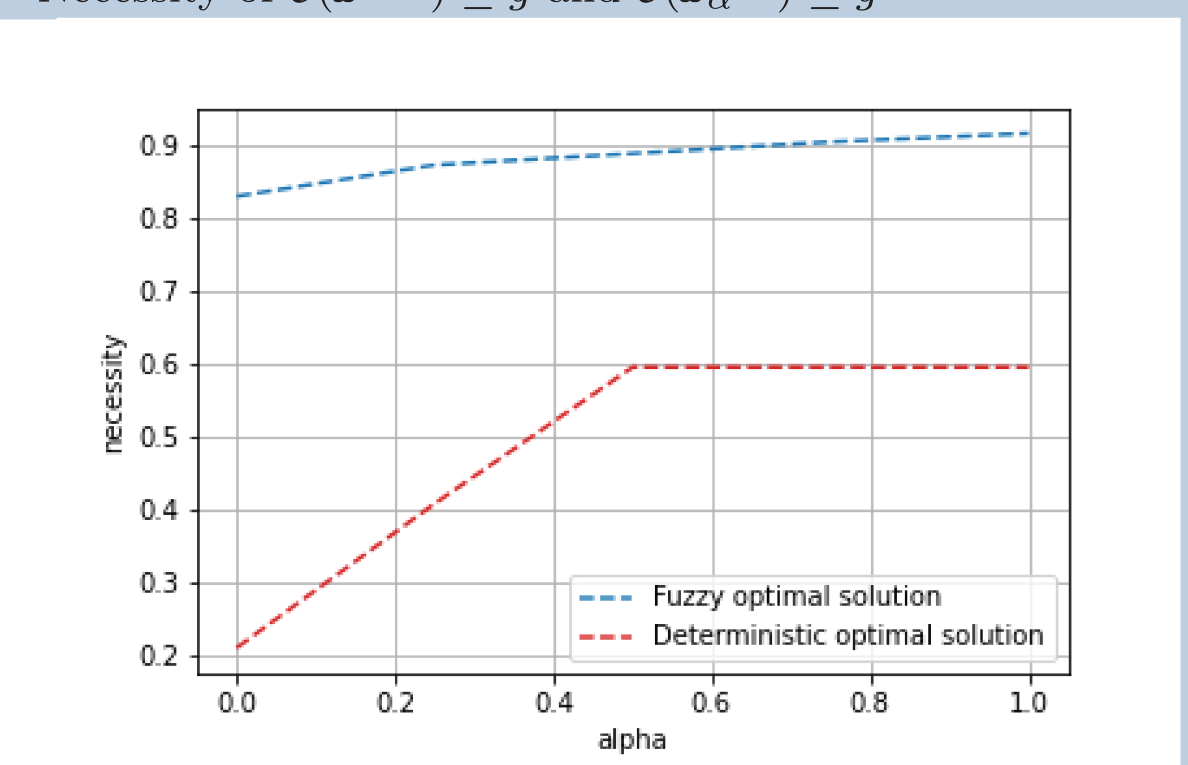
Illustration on example and experimental results

- We can see that globally the fuzzy robust approach moves the cumulative production to the left to be more robust to temporal uncertainty. Moreover when α decrease this phenomena increase too. It is due to the fact that when α decrease it is more possible that the demand of all periods deviated simultaneously. Hence the knowledge on dependencies moderate the conservatism of production.
- we can see that deterministic solution (solution for the nominal demand) can be risked event if we have high level of negatives dependencies ($\alpha = 1$) and naturally the risk increase if the dependencies become more positives.

Cumulative production



Necessity of $\mathcal{C}(\hat{\boldsymbol{x}}^{Opt}) \leq g$ and $\mathcal{C}(\boldsymbol{x}_\alpha^{Opt}) \leq g$



We can observe that when the dependencies (α parameter) increase the *Fuzzy robust* (solution for $\alpha = 0$) is less robust than the *alpha Fuzzy robust* hence over estimate the dependencies make lead to less robust solution. It is interested to note that with high dependencies the *Deterministic* solution is more robust than the *Fuzzy robust.*

Degree of necessity of different types of optimal solutions for $\alpha = 0.25$, $\alpha = 0.5$ and $\alpha = 1$