Possibility Propagation using Copulas

in a 3D Stereo-matching Pipeline

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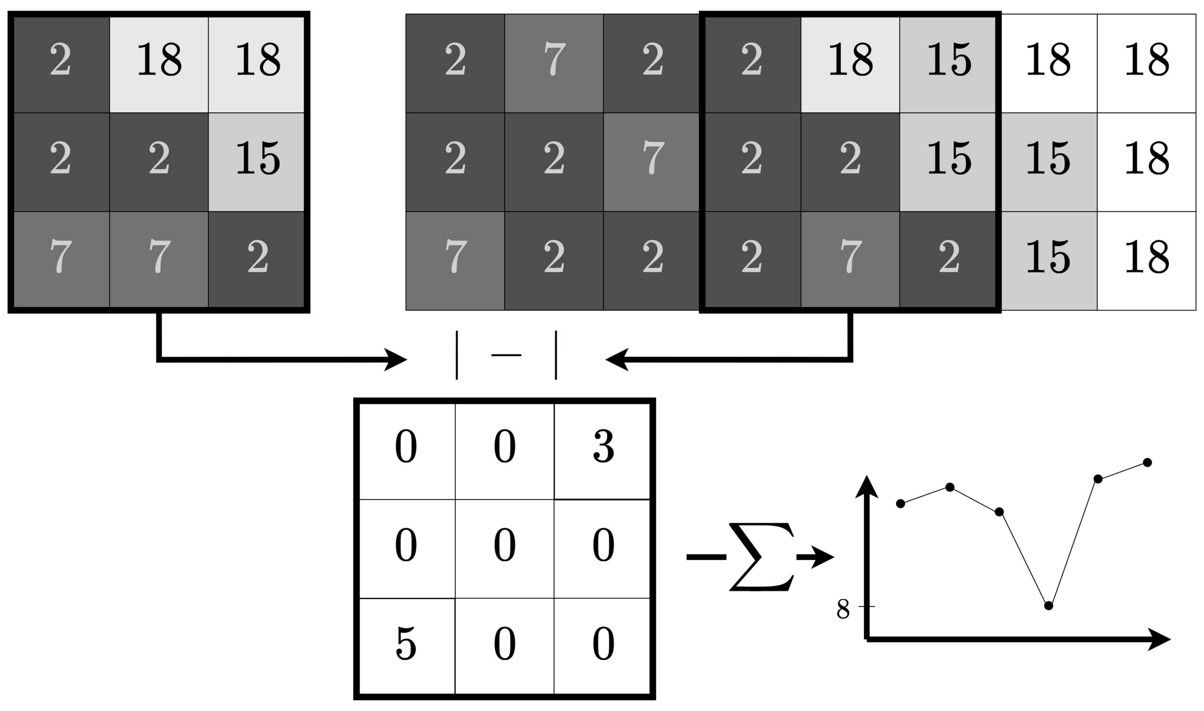
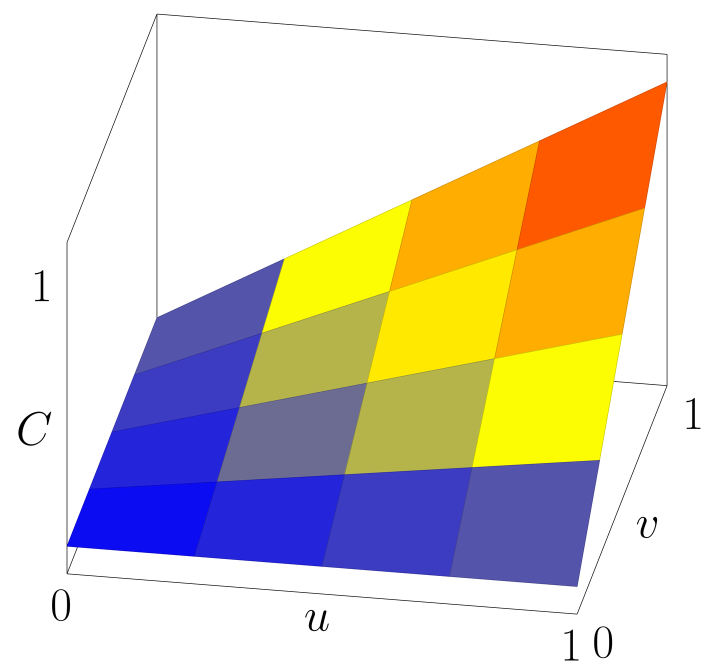
Problem Statement

Computing 3D models from pairs of images by means of stereo-matching is a complex problem. The crucial part consists in matching two pixels using a similarity function. How can we propagate the uncertainty to the similarity score when the uncertainty on pixel intensities is modeled by a possibility distribution, and the dependence between variables is a known copula?

Similarity Function



Copulas



A copula is a mapping $C : [0,1]^n \to [0,1]$ which can model any dependency between n Cumulative Distribution Functions.

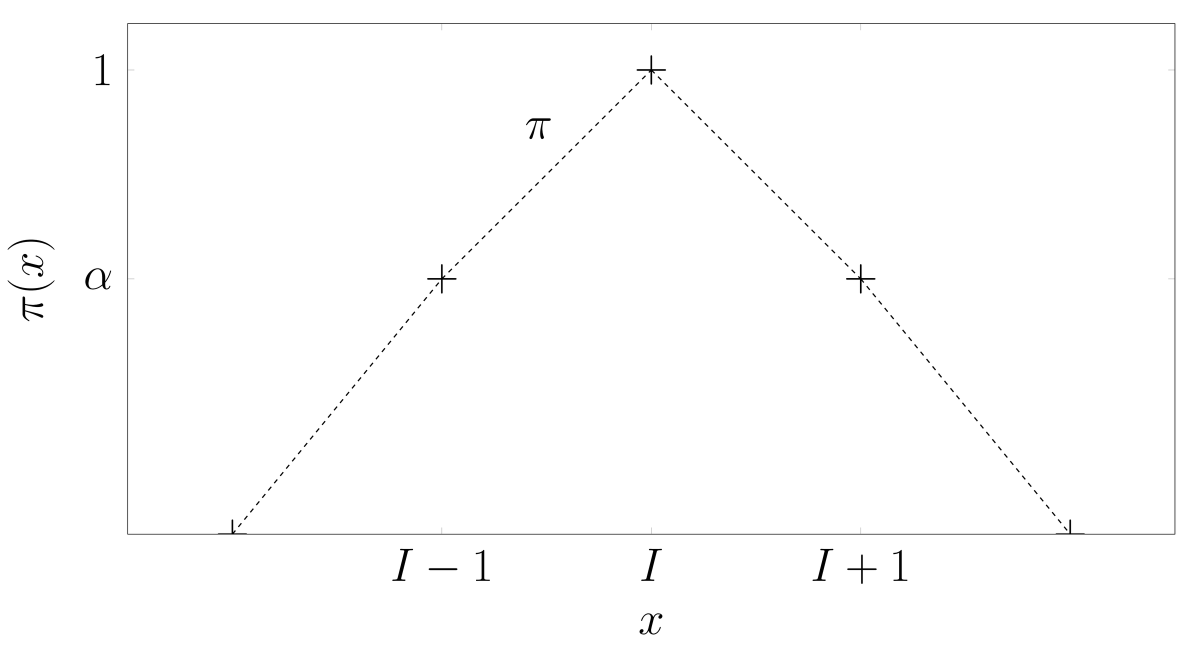
Sklar's theorem [1] states that any multivariate CDF G can be expressed using a unique copula C and its marginals F_i:

$$G(x_1,\dots,x_n) = C(F_1(x_1),\dots,F_n(x_n))$$

Conversely, any copula applied to n univariate CDFs correctly defines a n-dimension CDF.

Necessity Functions as Marginals

Because of sampling and digitization, the uncertainty regarding a pixel value is modeled by a possibility distribution centered around the observed value I. Possibility distributions define special types of belief functions called Necessity functions : $Nec(A) = 1 - \sup_{x\in A^c} \pi(x)$



Joining Belief Functions using Copulas

One way of aggregating Belief functions using a copula is by joining the masses of marginal belief functions using the H-volume of the copula [2]. In the bivariate case, writing $A_I = \sum_1^I m_X(a_i)$ and $B_J = \sum_1^J m_X(b_j)$, the H-volume is computed as follows:

$$\begin{aligned} m_{XY}(a_I, b_J) &= C(A_I, B_J) + C(A_{I-1}, B_{J-1}) \\ &- C(A_{I-1}, B_J) - C(A_I, B_{J-1}) \end{aligned}$$

The resulting mass function of a variable $Z = f(X,Y)$ is then:

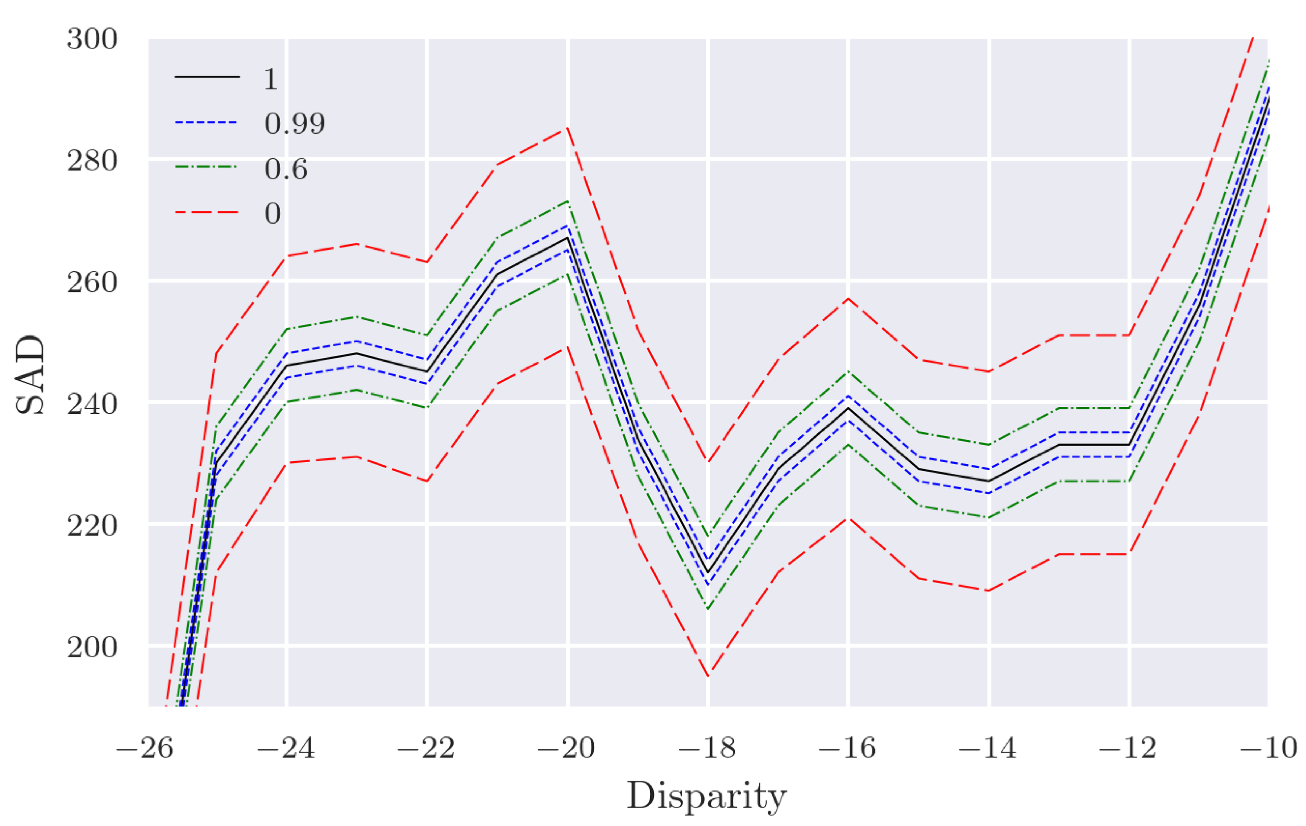
$$m_Z(z) = \sum_{\substack{a_I, b_J \\ f(a_I,b_J)=z}} m_{XY}(a_I, b_J)$$

and thus $Bel_Z(\mathcal{Z}) = \sum_{z\subseteq \mathcal{Z}} m_Z(z)$.

Faster computations

Here, the possibility distributions are symmetric, unimodal, and the similarity function can be monotone. It can be shown that in this case, joining Necessity functions with a Copula and propagating it using the function f yields a Necessity function. Adding to this the fact that $Nec_{XY} = C(Nec_X, Nec_Y)$ allows to easily compute the propagated focal sets and their bounds without computing the H-volume.

Computed Belief function



Monte Carlo Sampling

References

[1] M. Sklar. *Fonctions de Répartition À N Dimensions Et Leurs Marges*. Université Paris 8, 1959.
[2] Scott Ferson et al. *Dependence in probabilistic modeling, Dempster-Shafer theory, and probability bounds analysis*. en. Tech. rep. Oct. 2004.

Conclusion and Perspectives

- Copulas can be used to correctly propagate possibilities
- Computation can be reduced for symmetric possibility distributions
- This leads to a choice under uncertainty problem

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