Dynamic Precise and Imprecise Probability Kinematics

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Bayes' update is not always possible

- Bayes' rule $P(A \mid E) = \frac{P(A \cap E)}{P(E)} = \frac{P(E|A)P(A)}{P(E)}$ is arguably the most common updating rule for subjective beliefs

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  - Can be a very challenging task, for instance when E is not anticipated Jeffrey (1957, 1965, 1968)
- Evidence is not always propositional (i.e. it may not be possible to represent it as a crisp subset)
  - It is oftentimes uncertain or partial

Jeffrey's rule of updating

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- Generalizes Bayes' rule

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- DPK sits in between Bayes' and Jeffrey's rules
  - While it is built as a particular case of PK, it uses the empirical distribution to assign probabilities to the elements of the partition $\mathcal{E}$
  - To mechanize the procedure, it gives up the freedom of choosing the probability the agent feels correct to assign to the elements of $\mathcal{E}$

DPK properties

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  - Properties of successive partitions as more data are observed

Dynamic Imprecise Probability Kinematics

- We generalize DPK to deal with the agent facing ambiguity Ellsberg (1961); Gilboa and Marinacci (2013)
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- We study the convergence property of the DIPK rule

DPK/DIPK examples

- We give examples to show how to update subjective beliefs according to DPK and DIPK

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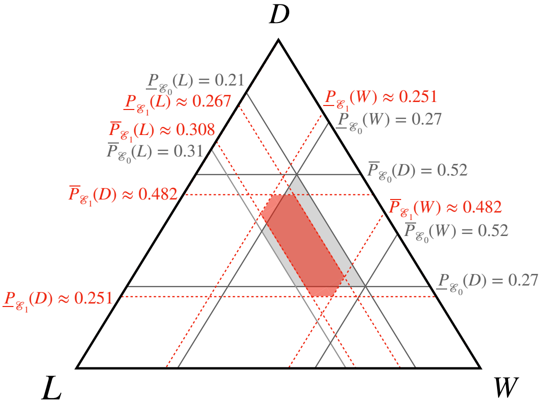

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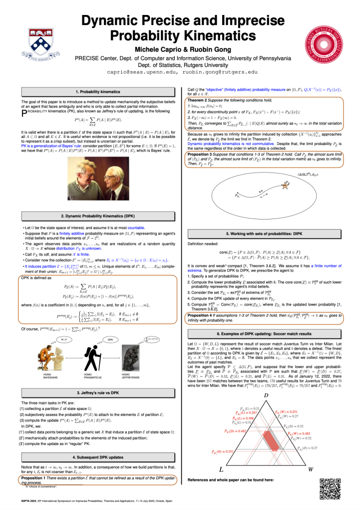

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