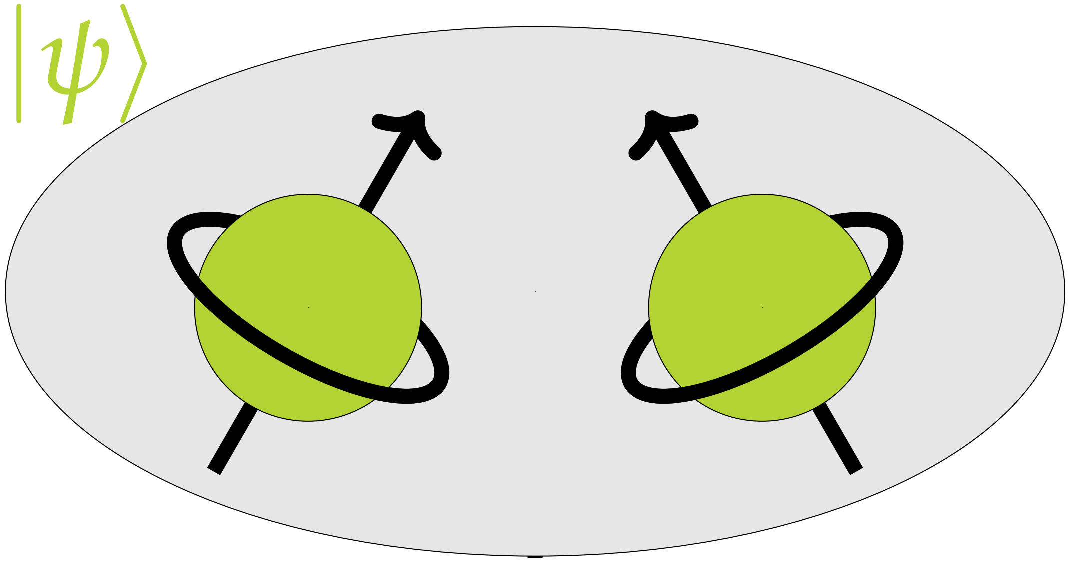
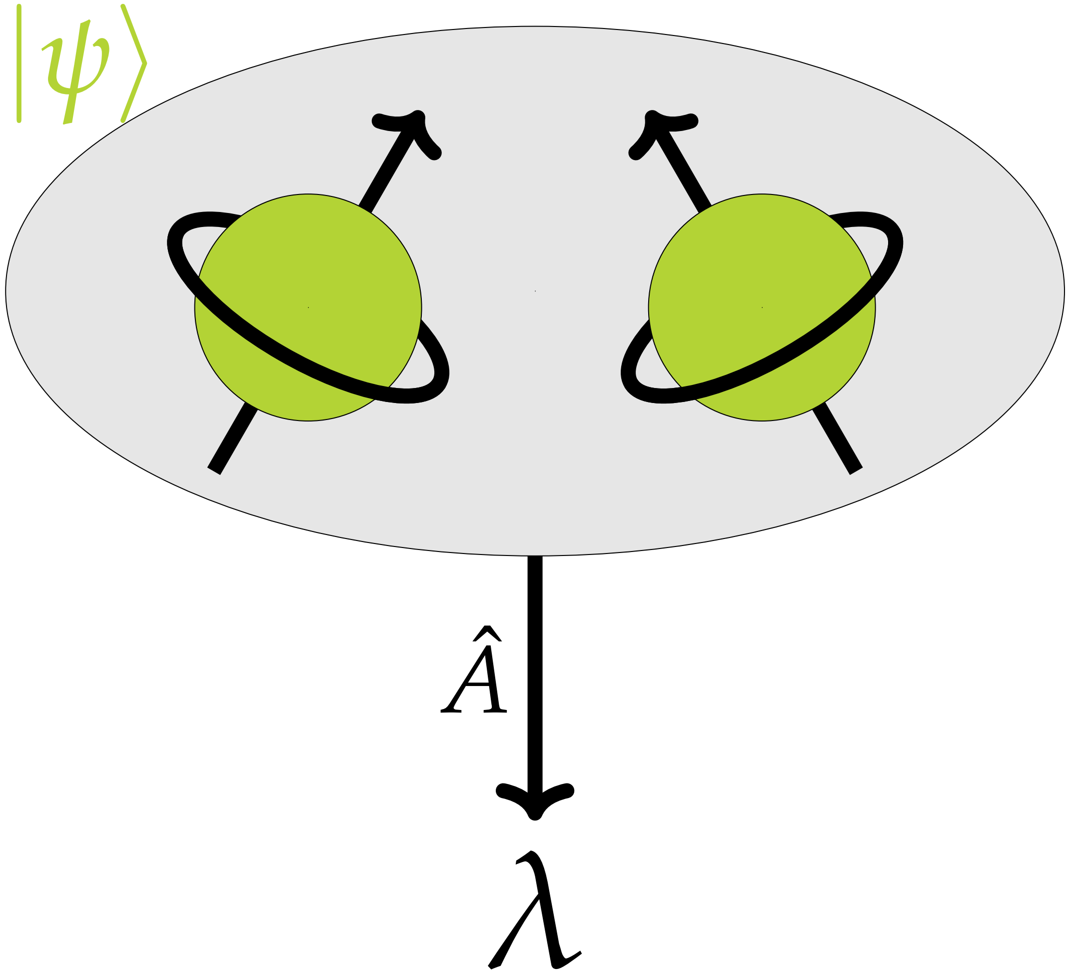
Indistinguishability through Exchangeability in Quantum Mechanics?



Quantum Mechanics

State $|\psi\rangle$:
element of a finite dimensional Hilbert space $\mathscr{H}$



Quantum Mechanics

State $|\psi\rangle$:
element of a finite dimensional Hilbert space $\mathscr{H}$

Measurement $\hat{A}$:
Hermitian operator on $\mathscr{H}$

Possible outcomes:
the eigenvalues λ of $\hat{A}$

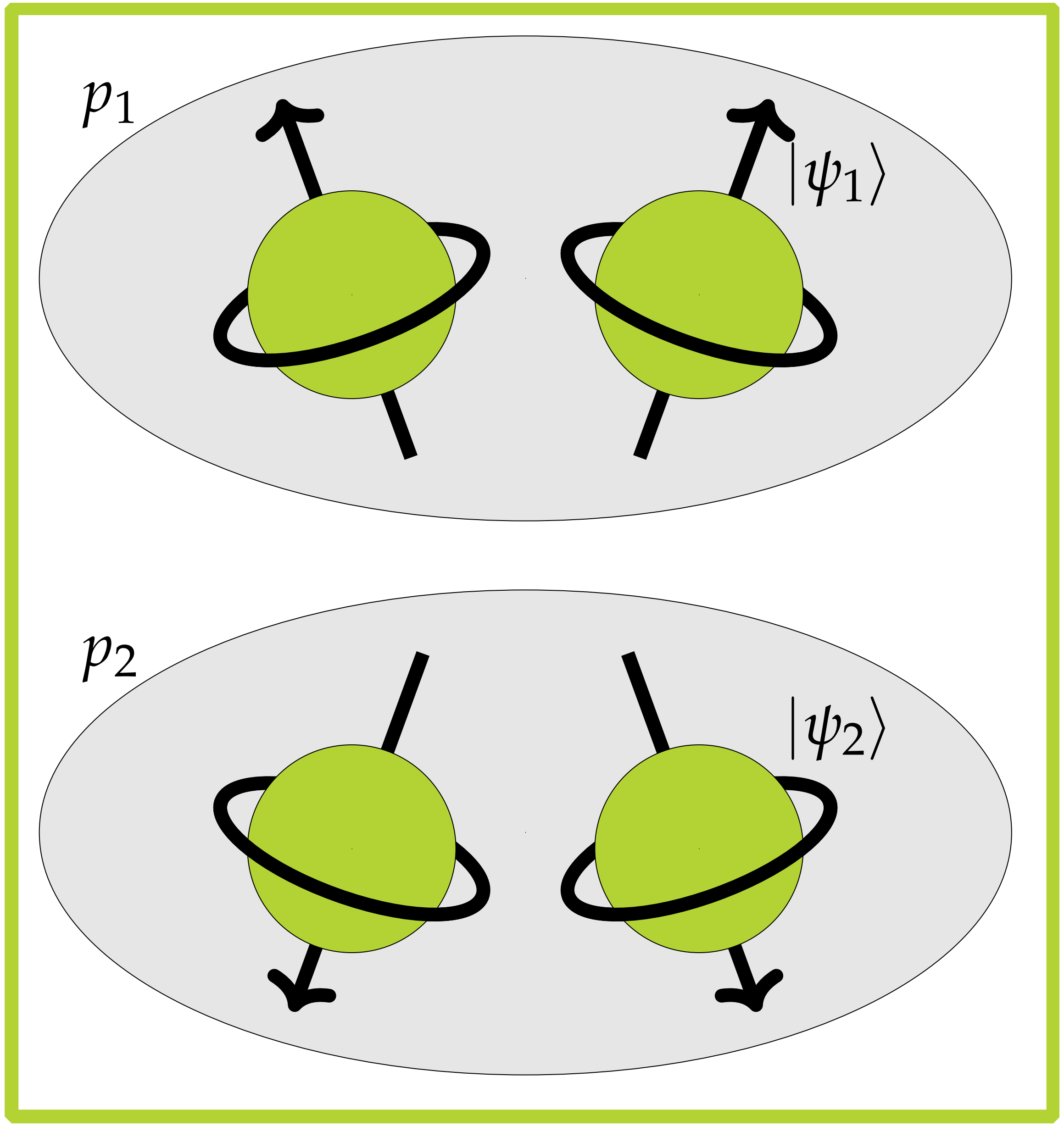
Uncertainty about the state

Density operator $\hat{\rho} := \sum_{k=1}^{r} p_k |\psi_k\rangle\langle\psi_k|$

With

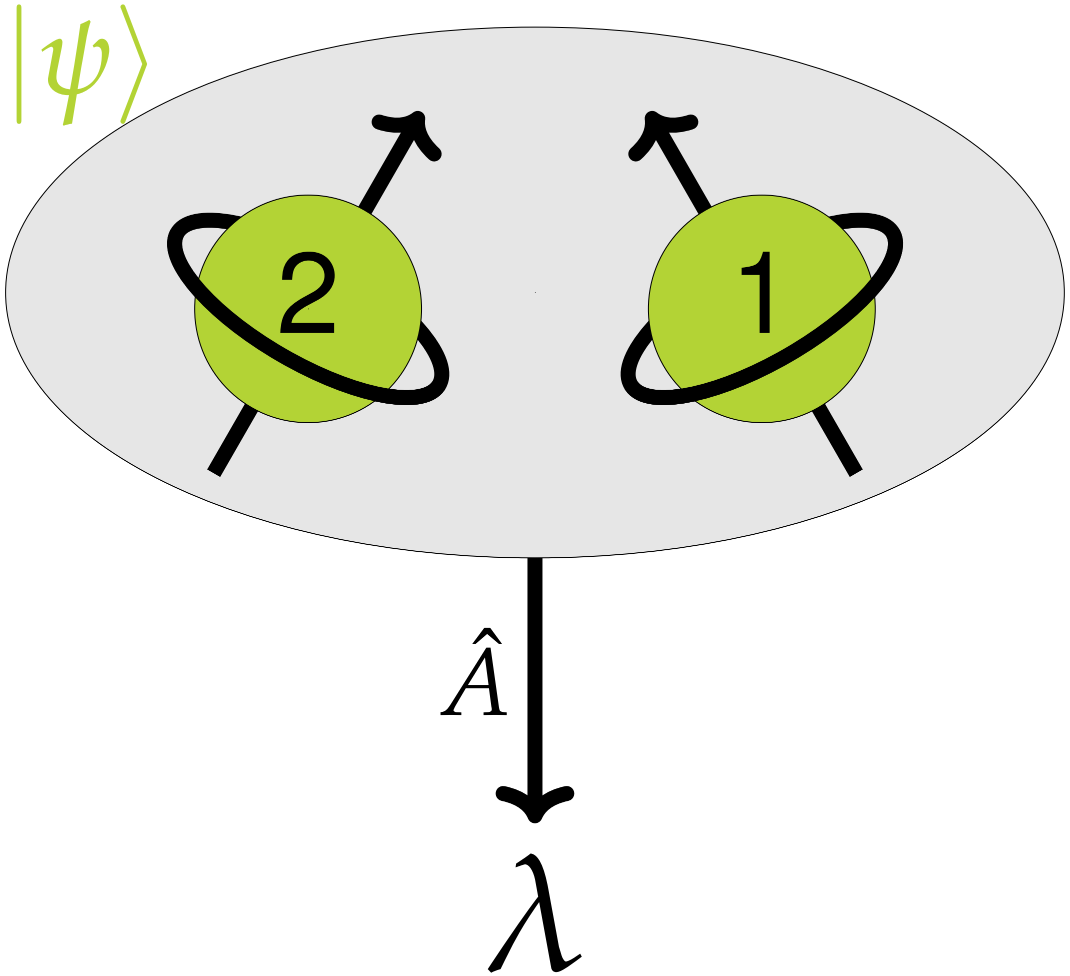
Probabilities $p_1, \ldots, p_r$

States $|\psi_1\rangle, \ldots, |\psi_r\rangle$



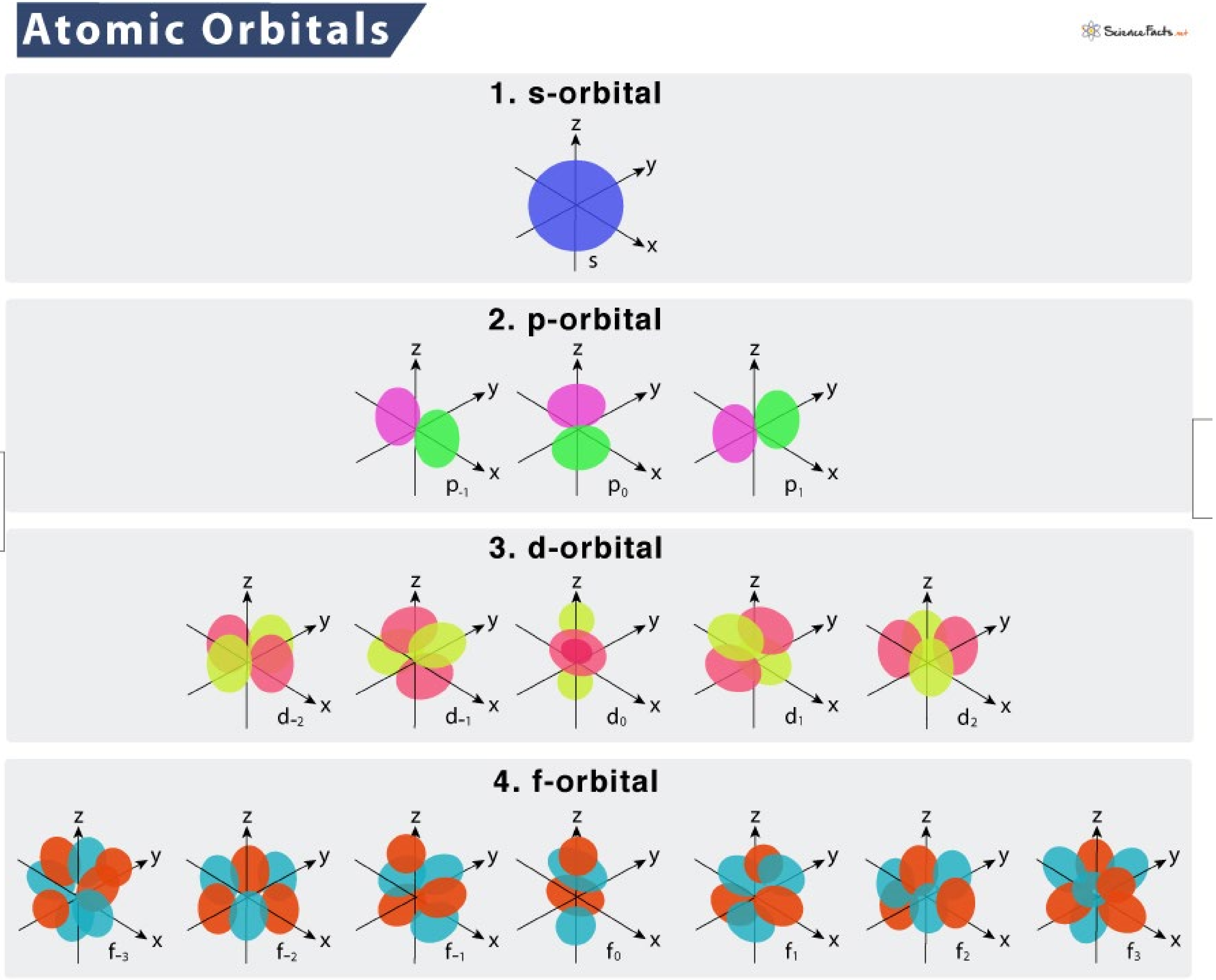
$$\hat{\rho} = p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2|$$

Indistinguishability

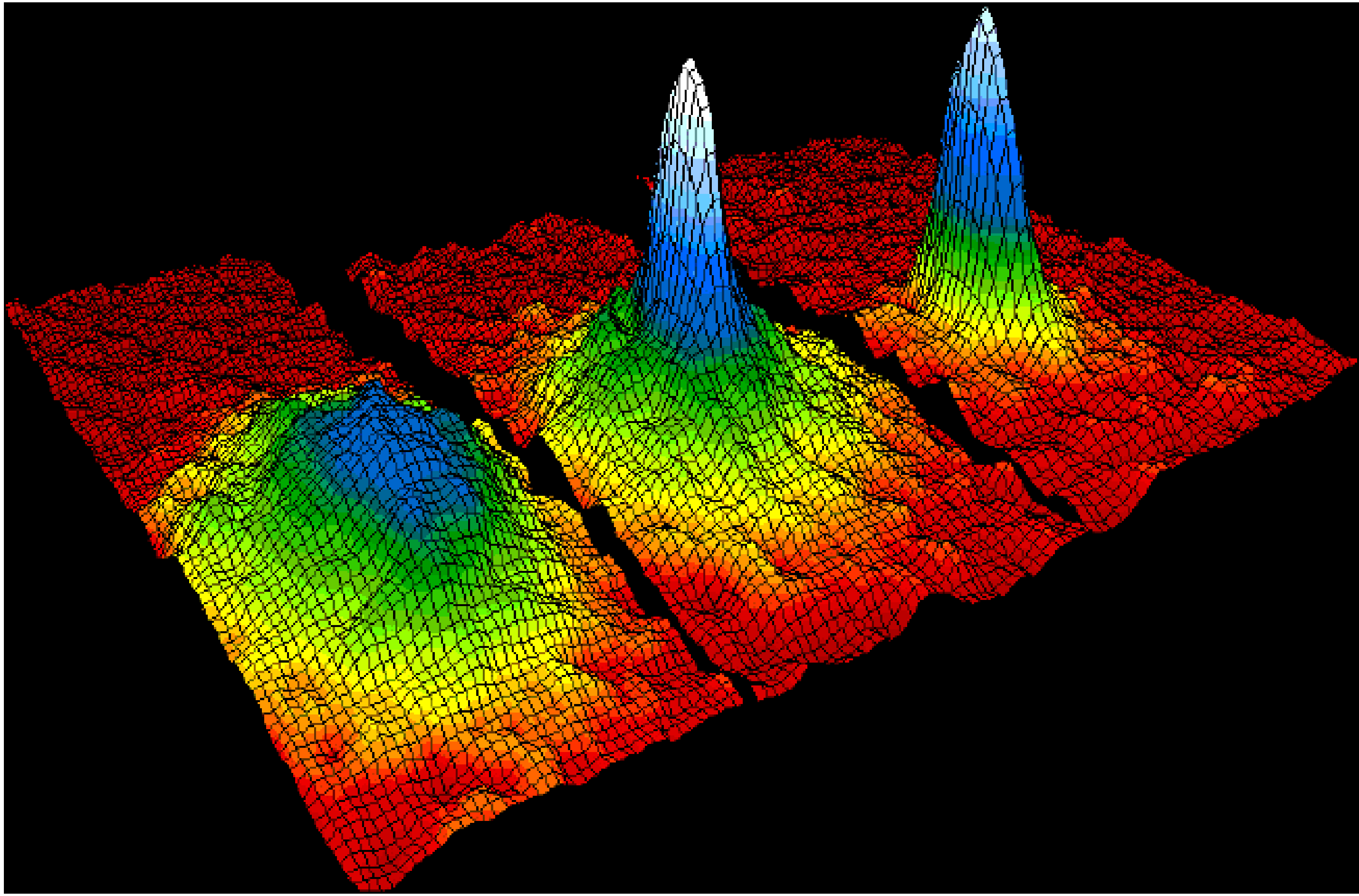


1 or 2?

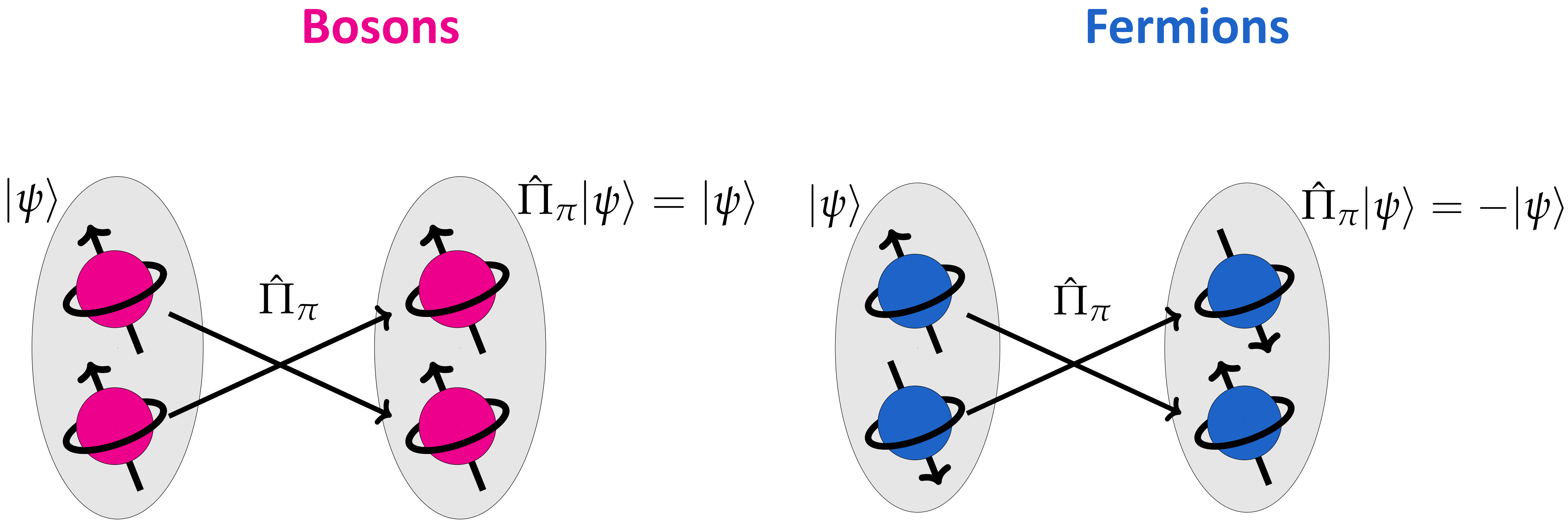
Fermi exclusion principle

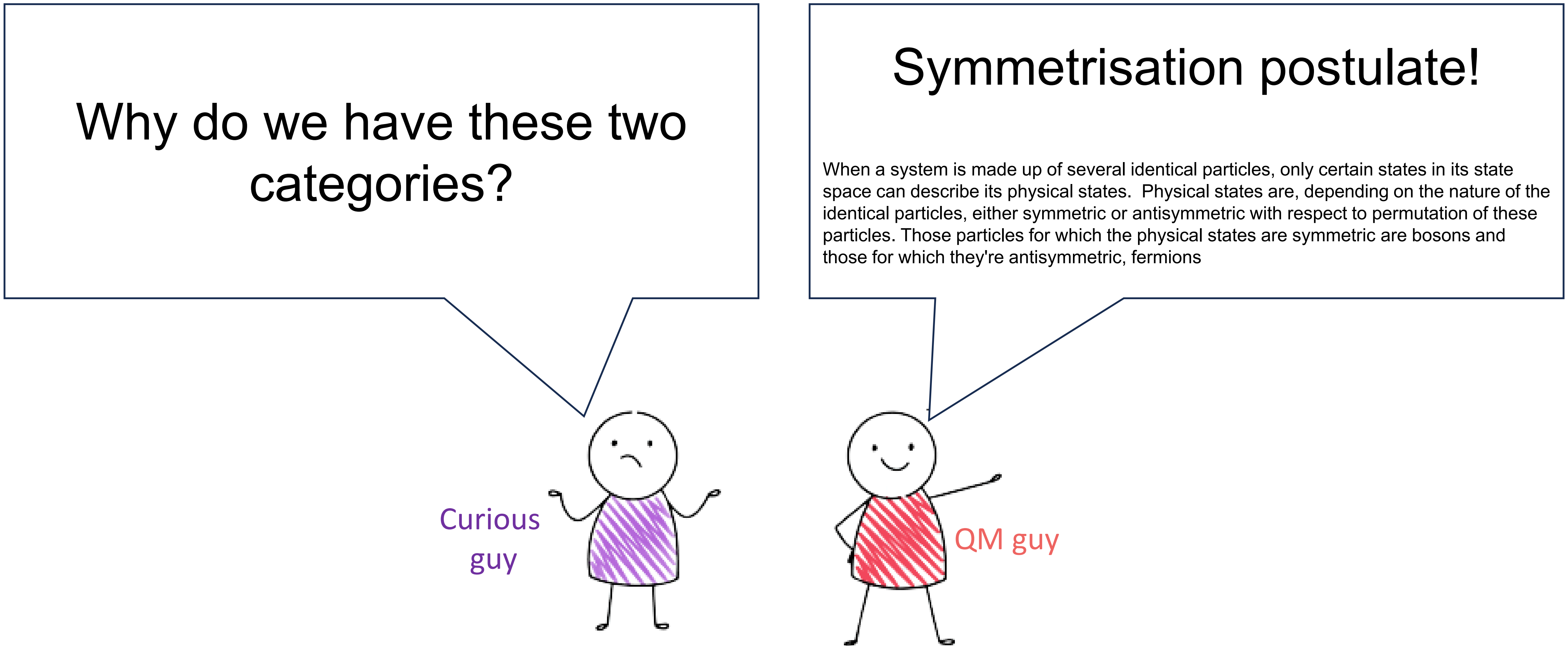


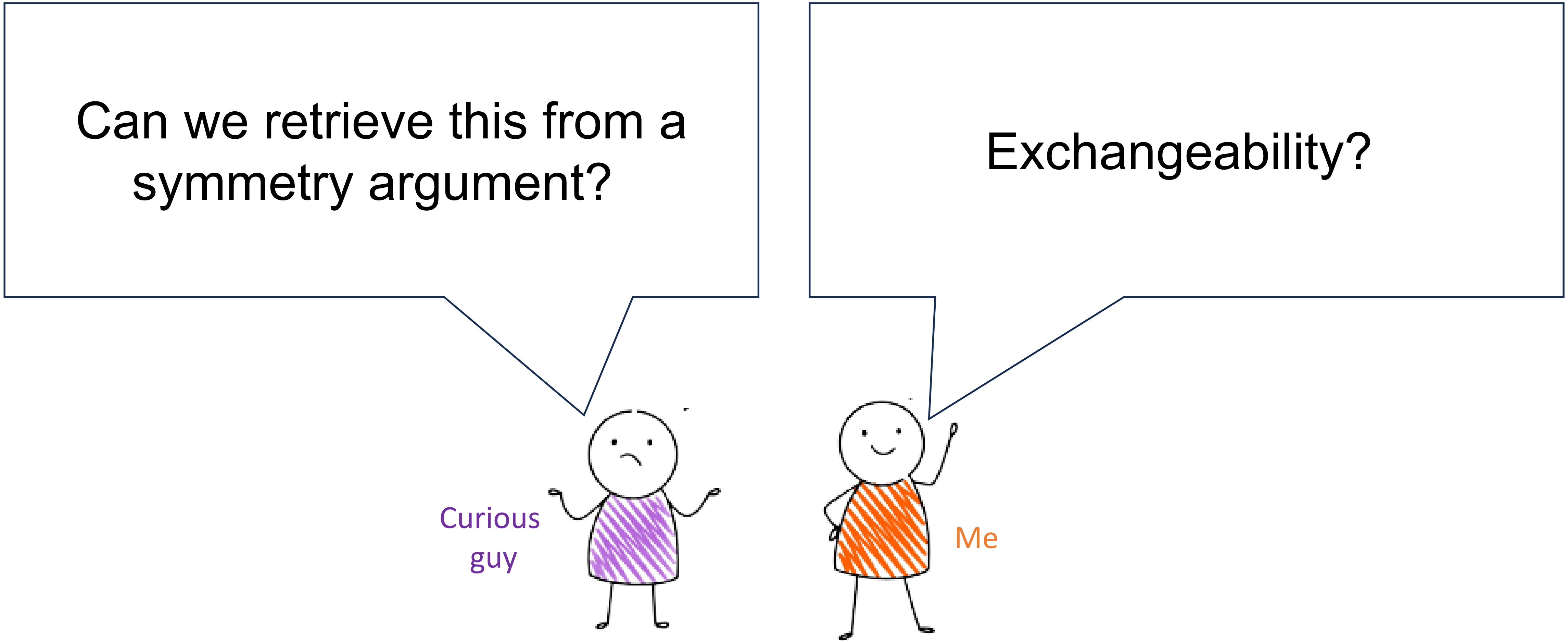
Bose-einstein condensate



Identical particles




Why do we have these two categories?
Curious guy
Symmetrisation postulate!
When a system is made up of several identical particles, only certain states in its state space can describe its physical states. Physical states are, depending on the nature of the identical particles, either symmetric or antisymmetric with respect to permutation of these particles. Those particles for which the physical states are symmetric are bosons and those for which they're antisymmetric, fermions
QM guy


Can we retrieve this from a symmetry argument?
Exchangeability?
Curious guy
Me

Exchangeability on?

Classical framework	QM equivalent
Probability distribution	Density operator
Set of probability distributions	Set of density operators
Set of desirable gambles	Set of desirable measurements

Set of density operators

Closed and convex ⇕ Up to boundary behaviour

Set of desirable measurements

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Check for updates

The Weirdness Theorem and the Origin of Quantum Paradoxes

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Abstract
We argue that there is a simple, unique, reason for all quantum paradoxes, and that such a reason is not uniquely related to quantum theory. It is rather a mathematical question that arises at the intersection of logic, probability, and computation. We give our 'weirdness theorem' that characterises the conditions under which the weirdness will show up. It shows that whenever logic has bounds due to the algorithmic nature of its tasks, then weirdness arises in the special form of negative probabilities or non-classical evaluation functionals. Weirdness is not logical inconsistency, however. It is only the expression of the clash between an unbounded and a bounded view of computation in logic. We discuss the implication of these results for quantum mechanics, arguing in particular that its interpretation should ultimately be computational rather than exclusively physical. We develop in addition a probabilistic theory in the real numbers that exhibits the phenomenon of entanglement, thus concretely showing that the latter is not specific to quantum mechanics.

Set of desirable measurements $\mathscr{D}$

Decision theoretic approach

~~probabilities~~

Decision-theoretic postulate 1
Let $\mathcal{E}$ be the eigenspace corresponding to an eigenvalue λ of the measurement operator $\hat{A} \in \mathcal{A}_{\mathrm{h}}$. Then $u_{\hat{A}}(|a\rangle) = \lambda$ for all $|a\rangle \in \mathcal{E}$ with $\langle a|a\rangle = 1$.

Decision-theoretic postulate 2
Let $\hat{A} \in \mathcal{A}_{\mathrm{h}}^n$ be an operator and let $\mathcal{B} = \{|a_1\rangle, |a_2\rangle, \dots, |a_n\rangle\}$ be a orthonormal basis of eigenkets of $\hat{A}$, where the eigenket $|a_i\rangle$ corresponds to eigenvalue λ_i for all $i \in \{1, 2, \dots, n\}$. Let $\pi : \{1, 2, \dots, n\} \to \{1, 2, \dots, n\}$ be any permutation of the indices. Now let $\hat{B} \in \mathcal{A}_{\mathrm{h}}^n$ be the operator with the same eigenkets $|a_1\rangle, |a_2\rangle, \dots, |a_n\rangle$, but now corresponding to the respective eigenvalues $\lambda_{\pi(1)}, \lambda_{\pi(2)}, \dots, \lambda_{\pi(n)}$. Consider the normalised kets $|\psi_{\hat{A}}\rangle, |\psi_{\hat{B}}\rangle \in \bar{\mathcal{H}}$ described by

$$|\psi_{\hat{A}}\rangle = \sum_{i=1}^{n} \alpha_i |a_i\rangle \text{ and } |\psi_{\hat{B}}\rangle = \sum_{i=1}^{n} \alpha_{\pi(i)} |a_i\rangle.$$

Then

$$u_{\hat{A}}(|\psi_{\hat{A}}\rangle) = u_{\hat{B}}(|\psi_{\hat{B}}\rangle).$$

Decision-theoretic postulate 3
Let $\hat{A} \in \mathcal{A}_{\mathrm{h}}^n$ be an operator with eigenvalue λ_i corresponding to eigenspace $\mathcal{E}_i$ with $i \in \{1, 2, \dots, p\}$. Now take $\hat{B} \in \mathcal{A}_{\mathrm{h}}^{n+m}$ as the operator with eigenvalues λ_i corresponding to eigenspaces $\mathcal{E}_i \otimes \mathcal{H}^m$ with $i \in \{1, 2, \dots, p\}$. For all $|\psi\rangle \in \bar{\mathcal{H}}^n$ and $|\phi\rangle \in \bar{\mathcal{H}}^m$, we then have that $u_{\hat{A}}(|\psi\rangle) = u_{\hat{B}}(|\psi\rangle \otimes |\phi\rangle)$.

Decision-theoretic postulate 4
If $\hat{A}, \hat{B} \in \mathcal{A}_{\mathrm{h}}^n$ are commutating operators, then $u_{\hat{A}+\hat{B}}(|\psi\rangle) = u_{\hat{A}}(|\psi\rangle) + u_{\hat{B}}(|\psi\rangle)$ for all $|\psi\rangle \in \bar{\mathcal{H}}$.

Decision-theoretic postulate 5
The utility function is continuous: let $\hat{A} \in \mathcal{A}_{\mathrm{h}}^n$ be any operator and $|\psi_1\rangle, |\psi_2\rangle, \dots \in \bar{\mathcal{H}}$ be any sequence of kets, then

$$\lim_{i \to +\infty} |\psi_i\rangle = |\psi\rangle \implies \lim_{i \to +\infty} u_{\hat{A}}(|\psi_i\rangle) = u_{\hat{A}}(|\psi\rangle).$$

Exchangeability through indifference

imate Reasoning

journal homepage: www.elsevier.com/locate/ijar

Exchangeability and sets of desirable gambles

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A B S T R A C T

Sets of desirable gambles constitute a quite general type of uncertainty model with an interesting geometrical interpretation. We give a general discussion of such models and their rationality criteria. We study exchangeability assessments for them, and prove counterparts of de Finetti's Finite and Infinite Representation Theorems. We show that the finite representation in terms of count vectors has a very nice geometrical interpretation, and

Set of desirable measurements $\mathscr{D}$ exchangeable if

$$\mathscr{D} + \mathscr{I} \subseteq \mathscr{D}$$

with $\mathscr{I} = \{\hat{A} - \hat{\Pi}_\pi^\dagger \hat{A} \hat{\Pi}_\pi \colon \hat{A} \in \mathscr{H}, \pi \in \mathbb{P}\}$

up to boundary behaviour

↓

Set of density operators

Set of density operators

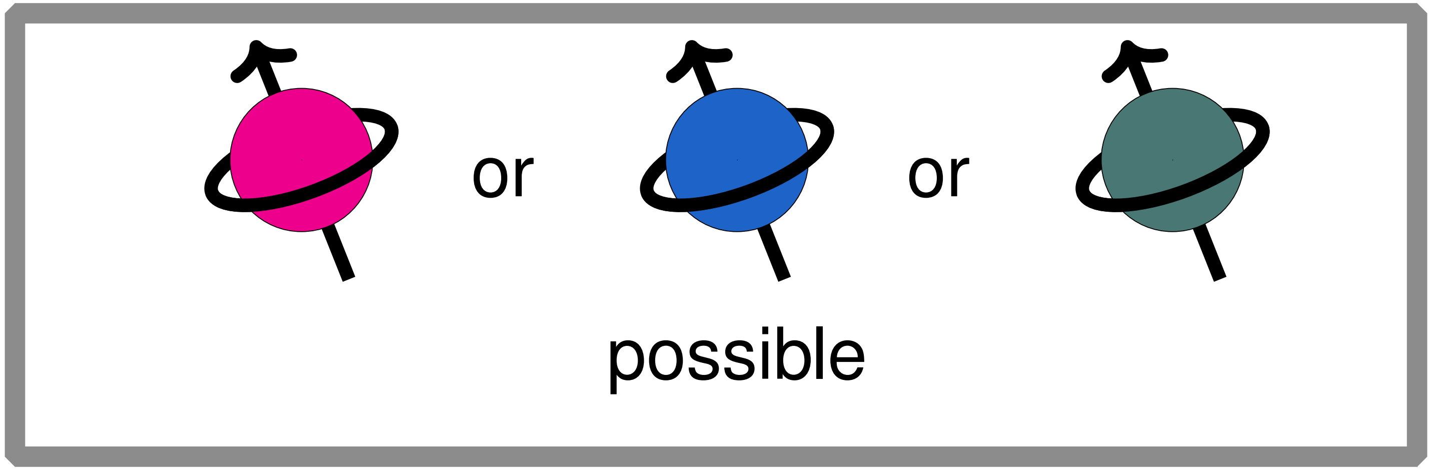
Permutation invariant

$$\hat{\rho} = \hat{\Pi}_{\pi}\hat{\rho}\hat{\Pi}_{\pi}^{\dagger} \text{ for all } \pi \in \mathbb{P}$$

$$\Updownarrow$$

Probabilistic mixing of bosonic, fermionic and para-particle density operator

$$\hat{\rho} = p_s\hat{\rho}_{\mathrm{s}} \quad + \quad p_a\hat{\rho}_{\mathrm{a}} \quad + \quad p_o\hat{\rho}_{\mathrm{o}}$$



Questions?

What are these para-particles?

Can we single out bosons and fermions?

Can we use count vectors?