HITTING TIMES FOR IMPRECISE CONTINUOUS-TIME MARKOV CHAINS

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Time-Homogeneous Continuous-time Markov Chains

- Models process P moving between states over time
  - Assumes Markov property: only depends on current state
  - Assumes time-homogeneity: no dependence on time
- Transition probabilities for a given time duration specified through rate matrix Q
- Hitting time: time until the process reaches some fixed subset of states A

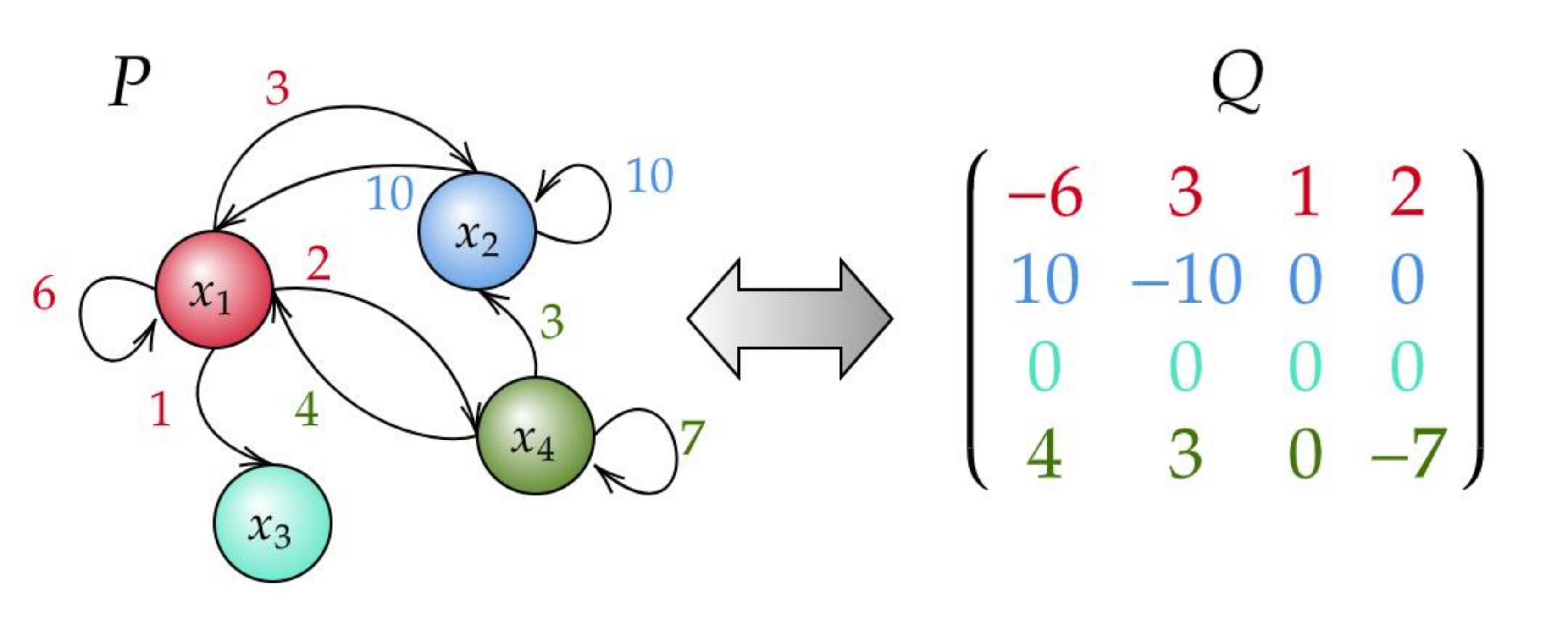

Figure 1: A time-homogeneous continuous-time Markov chain P and its rate matrix Q.

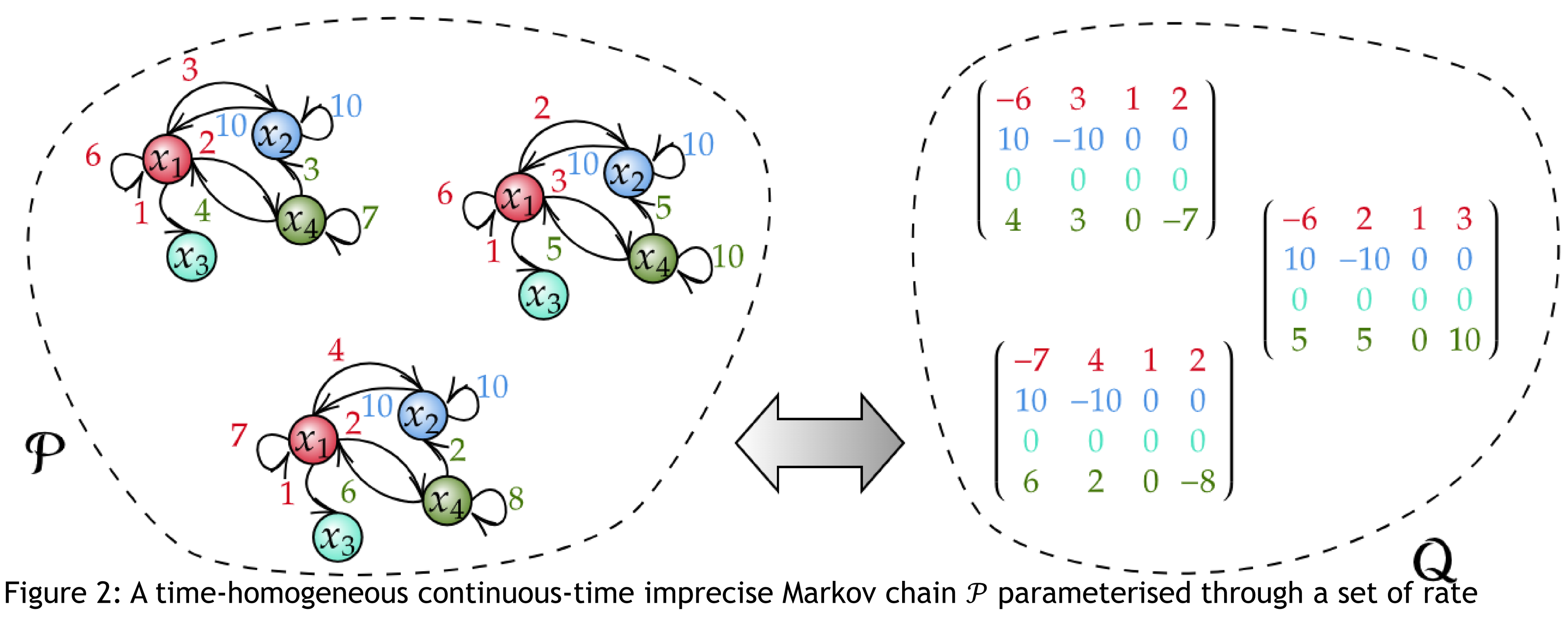

Figure 2: A time-homogeneous continuous-time imprecise Markov chain $\mathcal{P}$ parameterised through a set of rate matrices $\mathcal{Q}$.

Imprecise Markov Chains

- Consider instead of a single process, a *set of processes* $\mathcal{P}$, parameterised by a set of rate matrices $\mathcal{Q}$
- Can contain three types of processes:
  - Markovian & time-homogeneous
  - Markovian, but not necessarily time-homogeneous
  - General processes

→ *More robust to structural assumptions*: do not need to know whether the process is Markovian or time-homogeneous

→ *More robust to parameter uncertainty*: no longer need to precisely specify Q!

- Compute upper and lower bounds (upper and lower expectations) for desired inferences
  → Considers all processes in $\mathcal{P}$, without weighting them

State-of-the-Art [1]

- The upper and lower expected hitting times are the same for all three types of imprecise Markov chains
  → Can use computational inference tools suited for one specific type for all three types!
- Expected hitting times can be computed by solving a relatively simple non-linear system of equations

Problem:

- Previous results *currently assume that the probability of eventually hitting the set of states under consideration must be greater than 0*, which results in a bounded hitting time.

[1] T. Krak. "Computing Expected Hitting Times for Imprecise Markov Chains". In: International Conference on Uncertainty Quantification & Optimisation. Springer. 2020, pp. 185-205.

Our Contribution

- Divided the states into three classes.
- When starting in these classes:
  - $\mathcal{B}$: zero lower probability to reach A
  - $\mathcal{U}$: non-zero lower probability to reach A, but non-zero upper probability to reach $\mathcal{B}$
  - $\mathcal{Z}$: remaining states
- $\mathcal{Z}$ essentially behaves independently from $\mathcal{B}$ and $\mathcal{U}$ and this subspace satisfies the original assumption of Krak's work
  → Recovered previous results for both upper and lower expected hitting times
- $\mathcal{B}$ and $\mathcal{U}$ have been shown to have infinite upper expected hitting times for all three types of imprecise Markov chain
- $\mathcal{B}$ and $\mathcal{U}$ have been shown to have infinite lower expected hitting times for all three types of imprecise Markov chain, provided that a weaker assumption than Krak's assumption is met:
  → The reachability behaviour of all $Q \in \mathcal{Q}$ with respect to A is assumed to be similar

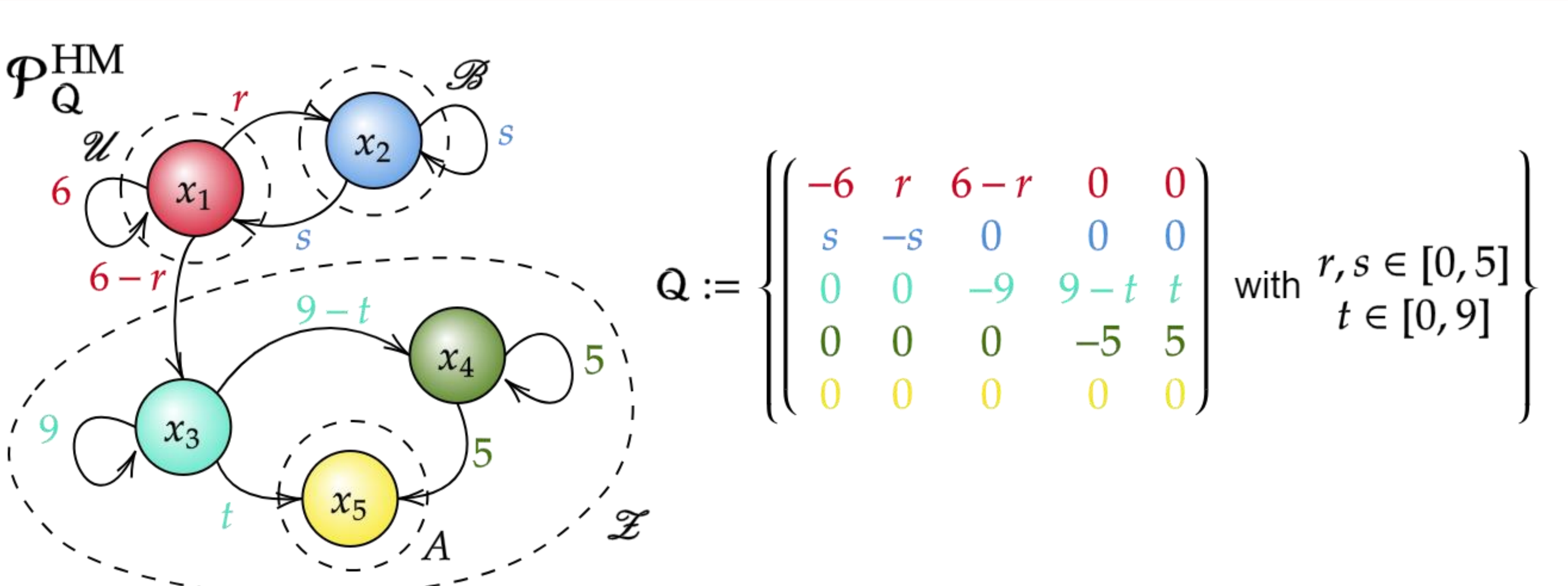

Figure 3: State classes